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IMPOSITION OPTIMALE SUR LE REVENU, CONTRAINTES D'INCITATION, CONTRAINTES DE PARTICIPATION

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INTRODUCTION GÉNÉRALE

L'impôt sur le revenu est l'un des instruments de redistribution privilégié dans la plupart des démocraties modernes. L'ampleur des prélèvements et la progressivité diffèrent cependant d'un pays à l'autre selon que l'équité ou l'efficacité sont plus ou moins valorisées. En effet, l'impôt sur le revenu constitue un instrument direct de redistribution des richesses entre les agents permettant de réduire les inégalités. Mais il est également une source de désincitation à l'effort pouvant conduire à un abaissement du niveau revenu national. Dans ce contexte se pose naturellement la question de la détermination du juste compromis entre équité et efficacité dans un pays donné, compte tenu de ses caractéristiques propres, mais également de l'environnement international dans lequel il s'inscrit. La théorie de l'imposition optimale sur le revenu répond précisément à cette interrogation.

Le cheminement logique qui préside à la détermination du meilleur impôt sur le revenu s'articule en trois temps. Il convient tout d'abord de s'entendre sur les objectifs poursuivis par le décideur public. Ces objectifs peuvent notamment résulter d'un processus démocratique. Il s'agit ensuite d'examiner les différents moyens d'atteindre les fins poursuivies, compte tenu des contraintes matérielles, institutionnelles, ou informationnelles. Enfin, il reste à identifier la meilleure solution réalisable, celle qui satisfait au mieux les objectifs au sein de l'ensemble des possibles. On suit ainsi la demarche préconisée par James Mirrlees : "A good way of governing is to agree upon objectives, discover what is possible, and optimize" (Mirrlees, 1986).

LA FORMULATION DU PROBLÈME D'IMPOSITION OPTIMALE SUR LE REVENU

En l'absence d'asymétrie d'information et autres imperfections et sous des hypothèses de convexités des préférences et des ensembles de production, le second théorème de l'économie du bien-être énonce que tout optimum de Pareto peut être obtenu par le libre jeu du marché, une fois que les transferts forfaitaires appropriés ont été réalisés. Les choix individuels ne sont pas alors distordus et un optimum de premier rang est atteint.

La formulation moderne du problème d'imposition optimale sur le revenu a pour point de départ la séparation fondamentale entre informations publiques et privées introduite par Mirrlees (1971, 1986). Mirrlees (1971) considère une économie fermée compétitive dont les agents diffèrent les uns des autres par leurs niveaux de productivité θ qui sont donnés et distribués continûment. Tous les individus ont les mêmes préférences sur un bien de consommation agrégé x et le travail ℓ, représentées par une fonction d'utilité de classe C^2 strictement concave $U(x, \ell)$, croissante en consommation et décroissante en travail. Le décideur public souhaite redistribuer les richesses des individus les plus talentueux vers les moins productifs. Cependant, il ne connaît que la distribution des compétences au sein de la population et ne peut observer le talent d'un agent particulier. Il est donc contraint de baser l'impôt sur le revenu brut $z = \theta\ell$.

Le décideur public se trouve dès lors confronté à un problème de révélation de l'information privée. Un agent productif peut en effet percevoir le même revenu qu'un agent moins talentueux en travaillant moins. Si l'imposition augmente avec le revenu, il peut alors être incité à réduire son revenu brut afin de payer un impôt plus faible tout en bénéficiant d'un loisir plus abondant. Dans cette hypothèse, sa consommation accrue de loisir est en partie financée par l'impôt, ce qui est à la fois contraire à l'équité et à l'efficacité. Les contraintes d'incitation ont pour mission d'empêcher un tel phénomène. Leur écriture se fonde sur la représentation alternative des préférences $U(x, \ell)$ dans le plan consommation/revenu brut, c'est-à-dire dans l'espace du contrat fiscal. Appelons $u\left(x\left(\theta'\right), z\left(\theta'\right); \theta\right) := U\left(x\left(\theta'\right), z\left(\theta'\right)/\theta\right)$ l'utilité d'un individu de compétence θ gagant un revenu brut $z\left(\theta'\right)$ dont il retire un revenu net $x\left(\theta'\right)$. La différence entre le revenu brut $z\left(\theta'\right)$ et le revenu net $x\left(\theta'\right)$ correspond à l'impôt payé $T\left(z\left(\theta'\right)\right)$. La mise en place d'un barème fiscal consiste à définir un mécanisme revenu brut/revenu net incitatif (x, z), c'est-à-dire tel que les agents ont intérêt à révéler leur information privée lorsqu'il est mis en œuvre. Les conditions d'incitation s'écrivent donc :

$$u(x(\theta), z(\theta); \theta) \geq u\left(x\left(\theta'\right), z\left(\theta'\right); \theta\right) \text{ pour tous les couples } \left(\theta, \theta'\right) \text{ possibles.} \tag{1}$$

L'utilité que procure à l'individu de type θ le couple revenu brut/revenu net $(x(\theta), z(\theta))$ doit être comparée à l'utilité obtenue pour tous les autres couples $\left(x\left(\theta'\right), z\left(\theta'\right)\right)$. Ces comparaisons devant être réalisées pour tous les types, les conditions d'incitation correspondent à une double infinité de contraintes dont le maniement difficile faisait obstacle à la résolution du problème d'imposition optimale découvert par Vickrey (1945).

La clé de résolution développée par Mirrlees (1971) se fonde sur la construction d'une représentation alternative des contraintes d'incitation à l'aide d'une condition d'intersection unique. Cette condition de Spence-Mirrlees repose sur l'idée qu'un agent plus compétent qu'un autre a besoin de fournir moins d'effort que ce dernier s'il souhaite accroître son revenu brut. En d'autres termes, un agent talentueux exige une augmentation de consommation inférieure à un agent moins talentueux lorsqu'il fournit un effort supplémentaire lui apportant un euro de plus. La Figure 1 donne la traduction graphique cette hypothèse centrale. Les revenus brut et net sont respectivement représentés en abscisse et en ordonnée. La courbe BB' correspond à un barème fiscal donné. Les courbes d'indifférence II' et JJ' rassemblent chacune les combinaisons (x, z)

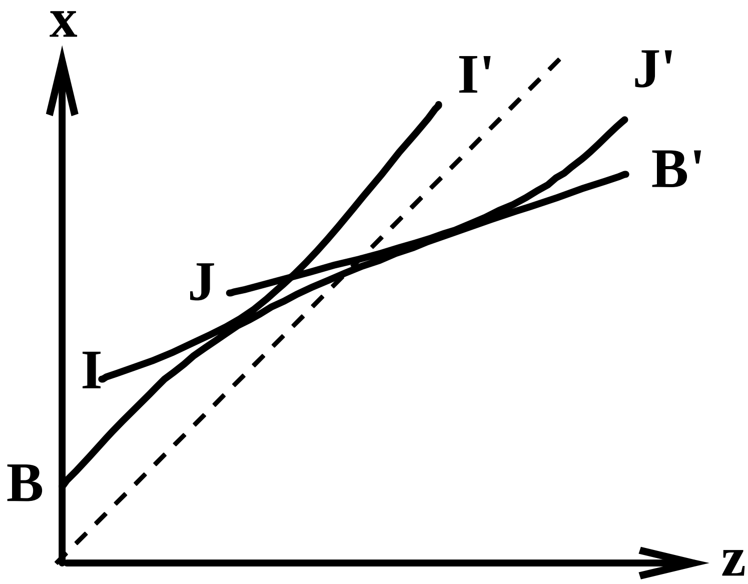

Fig. 1 – La condition d'intersection unique

qui procurent un niveau d'utilité particulier. En leur point d'intersection, il apparaît que la pente de II' est plus forte que celle de JJ'. A partir de ce point, augmenter le revenu net d'un euro tout en conservant un même niveau d'utilité est compatible avec un accroissement de revenu brut plus important le long de JJ' que de II'. L'agent θ_J dont les préférences sont représentées par JJ' trouve donc plus facile de produire plus de revenu brut que celui θ_I dont les préférences sont représentées par II'. Dès lors, retenir la condition de Spence-Mirrlees revient simplement à considérer qu'en tout couple revenu brut/revenu net, les agents ont des courbes d'indifférence d'autant moins pentues qu'ils sont compétents.

Un nouvel examen de la Figure 1 nous permet alors de découvrir quelles restrictions sur le barème fiscal sont imposées par la condition de Spence-Mirrlees. Supposons à cette fin que le barème fiscal (x, z) représenté par BB' satisfasse les conditions d'incitation. Sous la condition de Spence-Mirrlees, un agent plus compétent a des courbes d'indifférence moins pentues. Aussi son point de tangence avec le barème fiscal BB' ne pourra-t-il être à gauche de celui d'un agent moins compétent. En outre, chaque agent choisit une combinaison de revenus le long de BB'. Le barème fiscal apparaît alors comme l'enveloppe inférieure des courbes d'indifférence procurant aux individus l'utilité réalisable la plus grande. On obtient ainsi deux restrictions sur le barème. D'une part, les revenus net et brut ne doivent pas décroître avec la compétence. D'autre part, l'utilité indirecte doit augmenter avec la compétence, à un rythme donné par la dérivée de l'utilité par rapport à la productivité, à consommation et revenu constants. En particulier, un agent plus talentueux doit avoir un bien-être plus grand.

LES GRANDS RÉSULTATS D'IMPOSITION OPTIMALE SUR LE REVENU

Le modèle non-linéaire avec population continue

Un certain nombre de propriétés qualitatives ont été dégagées dans le cadre le plus général du modèle de Mirrlees (1971) avec une population continue. Tout d'abord, les taux marginaux d'imposition optimale sont non-négatifs (Mirrlees, 1971; Sadka, 1976; Seade, 1977; Ebert, 1992). Ils s'annulent pour les individus les plus productifs si la population est bornée (Mirrlees, 1971; Sadka, 1976; Seade, 1977; Ebert, 1992). Lorsque le loisir est un bien normal, les taux marginaux d'imposition augmentent strictement avec le revenu. Les individus plus riches paient alors un impôt strictement plus élevé (Seade, 1982). Un résultat parallèle à l'absence de distorsion en haut de la distribution des revenus est obtenu, en l'absence de bouchonnement (*bunching*), pour les individus les moins productifs dès lors que la borne inférieure de productivité est zéro. En revanche, en présence de bunching au bas de la distribution, le taux marginal d'imposition pour les individus les moins riches est strictement positif, ce qui donne lieu à un phénomène de trappes à pauvreté (Ebert, 1992).

Le modèle de Mirrlees (1971) a tout d'abord été étudié en ne tenant compte que des conditions du premier ordre pour l'auto-sélection. Celles-ci requièrent la non-décroissance de l'utilité indirecte avec la productivité, les conditions du second-ordre correspondant à la monotonie du revenu brut n'étant vérifiées qu'ex post. Brito and Oakland (1977) ont les premiers introduit explicitement les conditions du second-ordre dans la résolution du modèle et Ebert (1992) a fourni un exemple dans lequel ces conditions étaient violées par un barème satisfaisant pourtant toutes les autres conditions.

Du point de vue de la dérivation des formules d'imposition optimale, Piketty (1997), Roberts (2000) et Saez (2001) ont utilisé de petites réformes fiscales autour du barème optimal afin de mettre en lumière les différents effets sous-jacents. Dans le cas où les préférences individuelles sont quasi-linéaires en consommation, Diamond (1998) a obtenu une expression très claire des taux d'imposition optimaux qui résultent de la multiplication d'un facteur efficacité, d'un facteur éthique et d'un facteur démographique.

Les formules d'imposition optimale ont été implementées afin de dégager des indications de politiques publiques en retenant différentes spécifications de l'objectif social et des préférences individuelles (Mirrlees, 1971; Tuomala, 1990; d'Autume, 2000). Ces exercices révèlent la sensibilité du barème fiscal aux formes fonctionnelles retenues et soulignent que le barème fiscal non-linéaire n'est pas nécessairement proche d'un barème linéaire, contrairement à ce que suggéraient les simulations initiales de Mirrlees (1971).

Le modèle non-linéaire avec population discrète

Une spécification du modèle de Mirrlees (1971) dans laquelle la population est composée d'un nombre fini de types a été développée par Guesnerie and Seade (1982). Notons $\theta_1 < \cdots < \theta_i < \cdots < \theta_I$ les niveaux de productivité. Dans ce contexte, un barème d'imposition optimale correspond à un ensemble de coins et n'est précisément pas différentiable aux niveaux de revenu observés. Cependant, la différentiabilité des courbes d'indifférence permet de définir des taux marginaux implicites d'imposition. L'objectif du décideur public est donné par une fonction de bien-être sociale. Plutôt que de spécifier cette fonction, une hypothèse sur la nature de la redistribution socialement désirable est introduite. Il est ainsi supposé que redistribuer une certaine quantité de bien d'un individu compétent vers un individu moins compétent augmente le bien-être social en ignorant l'effet des contraintes d'incitation. Sous cette hypothèse et la condition de Spence-Mirrlees, toutes les conditions d'incitation (1) sont satisfaites lorsqu'aucun individu θ_i n'a intérêt à imiter le panier offert à son voisin moins productif le plus proche θ_{i-1}. A l'optimum, les distorsions sont minimisées et donc les contraintes d'incitation adjacentes vers le bas,

$$u\left(x\left(\theta_i\right), z\left(\theta_i\right); \theta_i\right) \geq u\left(x\left(\theta_{i-1}\right), z\left(\theta_{i-1}\right); \theta_i\right) \text{ pour } i = 2, \cdot, I, \tag{2}$$

sont toutes actives. On dit alors que l'optimum correspond à une chaine monotone simple à gauche. Dans ce cadre, on retrouve les mêmes propriétés en ce qui concerne le taux marginal d'imposition que dans le modèle continu. L'hypothèse de redistribution employée par Guesnerie and Seade (1982) a été relâchée par Röell (1985). Weymark (1986b) a étudié une version du modèle discret dans laquelle les individus ont des préférences quasilinéaires en loisir et les poids sociaux décroissent avec le talent. Il utilise la structure particulière qu'imposent les chaînes monotones à gauche pour dériver une forme réduite du problème d'imposition optimale qui ne dépend que des niveaux de consommation. Cette forme réduite permet la dérivation de résultats de statique comparative (Weymark, 1987) et la caractérisation des situations de bunching (Weymark, 1986a).

Les liens entre modèles discrets et continus sont difficiles à cerner en raison de la différence de traitement des conditions d'incitation. Hellwig (2007) a récemment dégagé des conditions sous lesquelles le modèle continu est équivalent à un problème d'imposition optimale dans lequel les agents de compétence θ_i ne peuvent avoir intérêt qu'à imiter des agents moins compétents. Ces conditions combinent l'hypothèse de Spence-Mirrlees et une version de l'hypothèse de redistribution désirable du modèle discret. Ce travail permet d'éclairer la structure commune des spécifications continues et discrètes.

Le modèle linéaire

La restriction de l'impôt sur le revenu à la linéarité apparaît attractive (Atkinson, 1997). L'impôt se compose alors d'un revenu minimum et d'un taux marginal unique, ce qui rend le barème fiscal beaucoup plus simple pour l'administration comme le contribuable. La linéarité de l'impôt présente également des avantages techniques dans la mesure où les conditions d'incitation n'ont pas à être introduites explicitement dans le programme de décideur public. Elle simplifie par conséquent de façon très significative la résolution du modèle de Mirrlees (1971). L'impôt linéaire optimal a notamment été étudié par Sheshinski (1972); Atkinson (1973, 1997); Romer

(1976); Dixit and Sandmo (1977); Helpman and Sadka (1978); Hellwig (1986) et Tuomala (1985) sur le plan théorique, une contribution essentielle consistant à arranger les conditions du premier ordre afin de les rendre plus transparentes.

Lorsque le loisir est un bien normal et que l'effet de substitution l'emporte sur l'effet revenu, Sheshinski (1972) a établi que l'impôt optimal linéaire purement redistributif comporte toujours un revenu minimum strictement positif, quand bien même serait-il très faible. Il s'agit d'un résultat très important par ses implications en termes de politiques publiques.

IMPÔT OPTIMAL SUR LE REVENU ET THÉORIE DES CONTRATS

Historiquement, la résolution par Mirrlees (1971) du problème d'imposition optimale sur le revenu marque la naissance de la théorie moderne des incitations. Le partage entre informations publiques et informations privées fonde en effet la théorie des incitations. Cette théorie étudie les règles et institutions induisant les individus à choisir un niveau d'effort adhéquat, au sens large, ou à révéler fidèlement des caractéristiques privées socialement pertinentes dont ils disposent (Laffont, 2003).

Le modèle principal-agent

Le modèle principal-agent dans lequel un mandant, appelé *principal*, souhaite déléguer l'accomplissement d'une tâche à un mandataire, appelé *agent*, forme le socle de la théorie des contrats (Laffont and Martimort, 2002). Du seul fait de la délégation, l'agent va avoir accès à des informations dont le principal ne dispose pas ou qu'imparfaitement. L'agent détient des informations privées pertinentes vis-à-vis de la conclusion du contrat de délégation. En retour, le principal ignore des caractéristiques importantes de l'agent lorsqu'il signe le contrat. Ces caractéristiques pertinentes constituent le *type* de l'agent. En raison de son incertitude sur le type de l'agent, le principal encourt un risque d'*anti-sélection*. D'autre part, la délégation d'une tâche expose le principal à une incertitude relative au comportement de l'agent. En effet, confier une tâche à autrui implique une perte au moins partielle de sa capacité de contrôle sur l'accomplissement de celle-ci. Le principal se trouve confronté à un problème d'*aléa moral*.

Bien entendu, l'agent n'est prêt à révéler son type et le comportement qu'il entend adopter que moyennant finance. Dès lors, le principal qui, croyant faire des économies, n'entendrait par principe débourser aucun denier pour obtenir une partie des informations privées des agents n'attirerait qu'incapables et tire-au-flanc. Les contrats conclus seraient alors totalement inefficaces. Le problème du principal est précisément de trouver le juste dosage entre l'extraction, coûteuse, de la rente informationnelle de l'agent et la préservation de l'efficacité.

A cette fin, le principal doit formuler un contrat pour lequel l'agent a spontanément intérêt à révéler correctement son type, ce qui se traduit par des *contraintes d'incitation* semblables à celles de la théorie de l'imposition optimale (1). Cependant, un bon contrat n'a pas pour seule caractéristique la révélation du type de l'agent. Des contrats peuvent par exemple être incitatifs, mais offrir des niveaux d'utilité si faibles qu'aucun agent n'accepte de les signer. Le principal qui souhaite voir la tâche déléguée accomplie est donc contraint d'offrir aux candidats potentiels un niveau de satisfaction suffisant compte tenu des alternatives qui s'ouvrent à eux. C'est l'idée restranscrite par les *contraintes de participation*. Un agent n'accepte un contrat que si la réalisation de celui-ci lui procure un bien-être supérieur à son *utilité de réservation*, correspondant à la meilleure option réalisable en dehors du contrat.

Le modèle d'imposition optimale au miroir de la théorie des contrats

Dans la théorie de l'imposition optimale, l'Etat-principal s'est vu déléguer la tâche de redistribuer les revenus par les citoyens-agents. Le principal n'observe ni le niveau d'effort ni le talent d'un agent particulier, mais connaît son revenu. Cependant, un agent de compétence donnée θ ayant décidé son niveau de revenu brut z n'a pas le choix de son niveau d'effort ℓ puisque, par définition, son revenu brut est donné par $z = \theta\ell$. Par conséquent, le lien déterministe entre revenu, compétence et effort permet de réduire le problème du principal à une incertitude sur le type, c'est-à-dire à un problème d'anti-sélection, formellement traduit à l'aide de contraintes d'incitation. Dans une économie fermée, tous les agents ont a priori la même utilité de réservation, que l'on peut normaliser à zéro. Si l'objectif social est redistributif, le décideur public ne va pas chercher à minimiser le niveau d'utilité $V(\underline{\theta})$ des individus les moins productifs. Par conséquent, à l'optimum, on aura $V(\underline{\theta}) > 0$. Comme les conditions d'incitation réquièrent une utilité indirecte non-décroissante avec la productivité, les contraintes de participation ne seront jamais actives, pour aucun agent. C'est pourquoi le modèle d'imposition optimale sur le revenu en économie fermée est un problème d'anti-sélection sans contraintes de participation mais avec une contrainte budgétaire. La contrainte budgétaire est souvent présentée comme le substitut des contraintes de participation en théorie de l'imposition. Elle ne joue cependant pas le même rôle. En outre, il y a autant de contraintes de participation que d'agents, alors qu'il n'y a qu'une seule contrainte budgétaire de l'Etat.

La mobilité internationale des agents complexifie l'arbitrage entre équité et efficacité. Les effets sur l'efficacité d'une augmentation du taux d'imposition à des fins d'équité jouent à présent par deux canaux. Le premier canal est similaire à celui déjà présent en économie fermée : une hausse des taux désincite au travail, ce qui se répercute sur le budget social par l'intermédiaire de la contrainte budgétaire de l'Etat. Le second canal est nouveau. Alourdir l'imposition sur le revenu réduit la rente de situation de l'agent qui émigrera si elle devient négative. Le départ de contribuables productifs grève les capacités de production nationales, c'est-à-dire la quantité de travail dans l'économie. L'Etat-principal doit donc examiner s'il convient de conserver les individus talentueux sur son territoire en leur offrant une compensation et, en particulier, jusqu'à quel prix il est socialement profitable d'acheter leur présence. Un problème de détermination de la taille optimale de la population résidente se trouve alors enchâssé dans le dilemme traditionnel entre équité et efficacité. L'introduction de contraintes de participation dans lesquelles l'utilité de réservation dépend du type des agents permet de prendre en compte cette dimension nouvelle.

L'interaction entre contraintes d'incitation et contraintes de participation a été étudiée dans des modèles de théorie des contrats qui ne s'intéressent qu'à la question de l'extraction de la rente informationnelle des agents au moindre coût (cf. Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995), et Jullien (2000)). Il apparaît dans ce cadre que les agents peuvent avoir avoir à la fois de bonnes raisons pour annoncer qu'ils sont de plus bas type et de plus haut type qu'en réalité. Un phénomène d'incitations inverses, appelée *incitations compensatoires*, apparaît aux côtés des incitations traditionnelles. Ceci traduit le fait qu'un agent peut décider d'imiter un autre individu en raison des opportunités externes de ce dernier. Lorsque les contribuables sont autorisés à voter avec leurs pieds, il apparaît intéressant d'introduire des contraintes de participation dépendant du type dans le problème d'imposition optimale de Mirrlees. Ce faisant s'opère un chassé-croisé singulier entre théorie de l'imposition optimale et théorie des contrats.

PLAN DE RECHERCHE

Les travaux présentés dans cette thèse s'appuient sur un cadre d'analyse commun, la théorie de l'imposition optimale sur le revenu développée par Mirrlees (1971). Du point de vue théorique, ils cherchent à mieux appréhender les conséquences des contraintes d'incitation dans un économie fermée (Chapitres 1 et 2) et la nature des interactions entre conditions d'incitation et de participation en économie ouverte (Chapitres 3, 4 et 5).

Le chapitre 1 étudie les conditions d'optimalité et les propriétés de statique comparative de la version discrète du modèle de Mirrlees en l'absence d'effets revenu sur l'offre de travail. Il utilise à cette fin la structure que placent les contraintes d'incitation sur la solution du problème d'imposition optimale lorsque la condition de Spence-Mirrlees est satisfaite. Une implication essentielle de la quasi-linéarité en consommation est que les poids sociaux de la forme réduite ne sont pas fonction des productivités individuelles, contrairement à Weymark (1987, 1986b,a). Ceci permet de séparer le rôle des niveaux de compétence de celui des considérations de nature éthique. Les optima sociaux intérieurs sont caractérisés par l'égalité entre un ratio des taux marginaux d'imposition implicites des agents adjacents et un poids social cumulé. La séparation claire entre productivités et poids sociaux est ensuite utilisée afin d'examiner l'effet sur l'optimum social d'une variation du talent individuel puis d'un changement des poids sociaux.

Le chapitre 2 étudie la proposition de barème fiscal linéaire en productivité développée par Kolm (2004) à l'aune de la théorie de l'imposition optimale. Il caractérise les objectifs sociaux bien-êtristes qui génèrent ce barème particulier lorsque la productivité est connaissance commune. Il montre que les poids sociaux décroissent alors linéairement par rapport au talent dès lors que l'optimum est intérieur. La possibilité de mettre en œuvre cette forme particulière d'impôt est ensuite explorée selon que le revenu brut et/ou le temps de travail sont observables par le décideur public. Une méthode pour implémenter les transferts de premier rang lorsque seul le revenu brut est observable est enfin proposée.

Le chapitre 3 présente les principaux modèles d'imposition optimale sur le revenu qui permettent de cerner les conséquences de la mobilité fiscale des agents les plus compétents. Il souligne en particulier que les modèles peu nombreux consacrés à cette question n'ont pas utilisé de contraintes de participation. Dans la mesure où cet instrument apparaît susceptible d'éclairer l'impact de l'exil fiscal sur les objectifs redistributifs des décideurs publics, les chapitres 4 et 5 sont consacrés à son introduction dans le modèle d'imposition optimale à la Mirrlees.

Les chapitres 4 et 5 développent un cadre commun d'analyse pour apprécier dans quelle mesure la menace potentielle d'émigration des agents les plus talentueux modifie la forme du barème optimal. Différents objectifs sociaux sont considérés, certains d'entre eux autorisant une variation de la taille de la population. Ceci permet notamment d'examiner la pertinence de l'idée assez largement répandue qu'il convient à tout prix de retenir les contribuables les plus talentueux quitte à abaisser significativement le niveau de leur impôt et donc la redistribution vers les moins productifs. Dans le chapitre 4, l'impôt optimal est restreint à la linéarité. Il apparaît que le bien-être moyen de la population résidente d'un pays peut augmenter sous l'effet du départ des agents les plus compétents. Le chapitre 5 examine si des résultats semblables apparaissent en l'absence de restrictions sur la forme de l'impôt. Dans ce cas, la non-monotonie de la rente résidentielle modifie la structure des comportements incitatifs. Un agent compétent peut éprouver la tentation d'imiter plus productif que lui afin d'afficher des opportunités externes plus séduisantes qu'il pourra monnayer en conséquence si l'Etat souhaite qu'il n'émigre pas. Un comportement d'imitation vers le haut s'ajoute par conséquent au comportement traditionnel d'imitation vers le bas. L'interaction entre contraintes de participation dépendant du type et conditions d'incitation modifie donc la structure des comportements mimétiques. On retrouve ainsi, en imposition optimale, les incitations croisées découvertes en théorie des contrats (Lewis and Sappington, 1989; Maggi and Rodriguez-Clare, 1995; Jullien, 2000). En raison du mimétisme bi-directionnel, le choix du barème optimal a des effets très subtils sur la taille de la population résidente. En particulier, il n'est pas évident que l'Etat-principal obtienne le bien-être social le plus grand en prévenant l'émigration des agents les plus talentueux y compris dans le modèle non-linéaire.

CHAPITRE 1

CONDITIONS D'OPTIMALITÉ ET STATIQUE COMPARATIVE DE L'IMPÔT NON-LINÉAIRE SUR LE REVENU

1.1. INTRODUCTION[1]

This article adds to literature that develops qualitative properties of solutions to screening problems. It analyses a finite population version of Mirrlees (1971)'s model, as in Guesnerie and Seade (1982), but concentrates on the special case where individual utility is separable in consumption and leisure, and linear in consumption. It provides a geometric characterization of the optimum allocation and derives comparative static results. The informational framework corresponds to Roberts's (1984) assignment uncertainty. The policymaker wants to redistribute income from the more to the less productive individuals. However, if the distribution of this parameter within the population is common knowledge, each agent's productivity is private information. Accordingly, the policymaker faces an adverse-selection problem and is restricted to setting a tax scheme based on gross earnings.

Merely assuming that individual preferences are concave and increasing both in consumption and leisure does not yield many general results. Indeed, the optimal tax structure is the product of different sorts of interacting influences. It basically depends on the skill distribution (Diamond, 1998; Saez, 2001), on the government's aversion to income inequality, reflected by the welfare

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weights in the social objective function, but also on the responsiveness of labour supply. Moreover, the way in which all these influences interact is affected both by the incentive-compatibility constraints and the tax revenue constraint, which restrict the possibilities for income redistribution. Because of the complexity of the relationship between the optimal tax schedule and the set of underlying parameters, further investigations must usually resort to numerical simulations (Tuomala, 1990). This is an unfortunate state of affairs because some features of the model are necessarily left somewhat obscure by computational approaches, which are very useful for they allows the optimal tax rates to be quantified, but are not ideally suited for shedding light on the economic intuition behind the results.

That is why more restrictive functional forms of individual preferences have been considered in order to get clear-cut general results. Lollivier and Rochet's (1983) observation that quasilinear-in-income preferences yield closed-form solution for the optimal income tax problem has proved particularly fruitful. Applying their basic insights to the finite population framework, Weymark (1986b) has derived a reduced-form optimal income tax problem, characterized bunching (Weymark, 1987) and provided comparative static results (Weymark, 1987). The obtained reduced form has consumption as only variable. The income distribution is then implied by the incentive compatibility constraints. Moreover, the reduced-form problem has the form of an unconstrained maximization of a weighted utilitarian social welfare function. This methodology has then been adapted to obtain comparative static properties for a model in which the government both designs an optimal income tax and provides a public good optimally (Brett and Weymark, 2004). In the same vein, Hamilton and Pestieau (2005) have examined the impact of changing individual productivity in an economy with two classes of agents where the government adopts a maximin or maximax objective function. With a continuous population, the tractability allowed by quasilinear-in-income preferences has also been exploited by Ebert (1992) to provide a complete example in which different types of individuals are bunched together, establishing that the first-order approach to Mirrlees (1971)'s model can be misleading, and by Boadway, Cuff, and Marchand (2000) and Boone and Bovenberg (2007) to characterize the optimum allocation.

Assuming quasilinear-in-income preferences offers technical advantages because they are linear with respect to the variable observed by the government and used as the tax base. This notably allows the reduced-form optimal income tax problem to have an explicit solution. There are however some limitations in the linear-in-income model. The first is that the disutility of labour is constant. Hence, when a price is varied, the change in individual consumption does only depend on the substitution effect while all income effects are absorbed by labour supply. The second is that the reduced-form problem is not as informative as it first seems. Indeed, the weights in the reduced form are a combination of the weights in the underlying welfare function and the levels of skill describing the population. This complicates the interpretation of the optimality conditions and of some comparative static results.

At first sight, working with quasilinear-in-consumption preferences is technically less tractable : the linearity with respect to the observable variable is lost, the role of the labour supply elasticity is more complex in determining the optimal tax and the reduced-form problem has no explicit solution. However, this kind of preferences are worth examining for at least three reasons. First, most of the empirical studies, though not all, give credence to small income effects relative to substitution effects as regards labour supply (Blundell, 1992; Blundell and MaCurdy, 1999). Accordingly, the case with no income effect on labour supply provides a relevant benchmark, which has been extensively used in the continuous population model since the work by Diamond (1998) (cf. Atkinson (1990); Piketty (1997); Salanié (1998); d'Autume (2000); Boadway and Pestieau (2007); Saez (2001, 2002)). Second, from the theoretical viewpoint, assuming that all income effects are absorbed by consumption is a more satisfying assumption. Otherwise, the optimal tax schedule is independent of the labour response (Boadway, Cuff, and Marchand, 2000). Third, the comparative static properties of the optimal non-linear income tax problem could differ significantly from those obtained under quasilinear-in-income preferences.

This paper exploits the fact that, when some redistribution from the more to the less productive individuals is desirable, all conditions for incentive compatibility reduce to pairwise comparisons of utility levels between adjacent individuals (Guesnerie and Seade, 1982; Röell, 1985; Hellwig, 2007). This places structure on the optimal allocation and allows the derivation of a reduced-form income tax problem involving only the allocation of gross incomes within the population. This reduced form can be seen as a special case of Chambers (1989) "concentrated" objective function derived for separable preferences. The first implication of the linearity in consumption is that the welfare weights in the reduced form are not a function of the individual skill levels. This permits skills to be separated from underlying welfare considerations, which gives greater insight into the characterization of the optimum and its comparative properties. In particular, the trade-off between equity and efficiency is very transparent when the optimum is interior : at each observed gross income level, a wedge involving the ratio between the marginal tax rate of the individual for whom the bundle is designed and that of his nearest more productive neighbour, which reflects efficiency, must be equal to a cumulative social weight. This wedge allows a very simple geometric characterization of the optimal allocation in the absence of bunching and plays an important part in the comparative static analysis. It is closely related to the single-crossing property which corresponds to a restriction on its sign, whilst the optimality conditions restrict its magnitude. This observation is novel in the literature and is potentially useful for other kinds of screening problems employing the single-crossing property. The second significant result concerns the effect of a change in the skill level of an individual, which had not been addressed in the literature. In Mirrlees model, productivity is the sole parameter of heterogeneity within the population and the fundamental source of the self-selection problem since it basically conditions the effort a given individual has to undertake when he chooses to mimic everyone else. It is shown that a slight alteration in the skill level of any individual has the surprising effect of only affecting the implicit marginal tax rates and pre-tax incomes of himself and his nearest less productive neighbour ; the implicit marginal tax rate of no other consumer is modified. The last set of results examines in which ways the optimal allocation is affected by changes in the social weights. Since our reduced form has not to use skill-normalized social weights as in Weymark (1987), our comparative static results isolate the impact of varying the redistributive tastes of the policy-maker.

The paper is organized as follows. Section 2 sets up the model. Section 3 derives the reduced form of the optimal non-linear income tax problem and provides a geometric characterization of the optimal allocation. Section 4 examines the comparative statics of the solution to the optimal income tax problem. Section 5 concludes.

1.2. THE MODEL

The population consists of $I \geq 2$ individuals, indexed by $i \in \mathcal{I} := \{1, \ldots, I\}$. There are two goods, consumption and leisure. Units of the consumption good are chosen so that one unit costs one euro. Person i's consumption and labour supply are denoted x_i and ℓ_i, respectively. The economy is competitive, with constant-returns-to-scale technology ; so person i's wage rate is fixed and equal to his productivity θ_i. For convenience, only one person has a given productivity level. Individuals are thus indexed in terms of productivity. This simplification is not particularly restrictive as the distance between two productivity levels is free to vary. Without loss of generality, the vector of productivities $\theta := (\theta_1, \ldots, \theta_I)$ is taken to be monotonically increasing,

$$0 < \theta_1 < \ldots < \theta_I. \tag{1.1}$$

An individual with productivity θ_i working ℓ_i units of time has gross income

$$z_i := \theta_i \ell_i, \qquad i \in \mathcal{I}. \tag{1.2}$$

All individuals have the same preferences over consumption and leisure, represented by the utility function $U : \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}$,

$$U(x_i, \ell_i) := \gamma x_i - v(\ell_i), \qquad i \in \mathcal{I}, \tag{1.3}$$

where $\gamma \in \mathbb{R}_{++}$ is the marginal utility of money.[2] It is assumed that the disutility of labour $v : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a $\mathcal{C}^3$-function which satisfies $v' > 0$, $v'' > 0$, $v''' > 0$, $v(0) = 0$ and $\lim_{\ell_i \rightarrow \infty} v'(\ell_i) \rightarrow \infty$. The assumption $v''' > 0$ means that the marginal disutility of labour is convex. Hence, providing a marginal unit of labour time becomes increasingly unattractive when ℓ_i goes up, which seems quite reasonable.[3]

By (1.2), the utility function (1.3) can be rewritten as $U(x_i, \ell_i) = U(x_i, z_i/\theta_i)$. Individuals

[2] Similarly, it is typically assumed that $\ell_i \in \mathbb{R}$ in the quasilinear-in-income version of Mirrlees's model (Lollivier and Rochet, 1983; Weymark, 1986b, 1987).

[3] This assumption plays an important role in the analysis. It ensures that the reduced-form optimal income tax problem has a unique solution and also appears in some comparative static results. The role of third derivatives in comparative static exercises has been emphasized by the literature devoted to risk and uncertainty. For instance, see Laffont (1989); Kimball (1990); Menegatti (2001).

have therefore *personalized utility* functions $u : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$ in the gross-income/consumption space,

$$u(x_i, z_i; \theta_i) := \gamma x_i - v(z_i/\theta_i), \qquad i \in \mathcal{I}. \tag{1.4}$$

The *marginal rate of substitution* $s(z_i; \theta_i)$ of person i at the (x_i, z_i)-bundle is independent of his consumption level, with

$$s(z_i; \theta_i) := -\frac{u'_{z_i}(x_i, z_i; \theta_i)}{u'_{x_i}(x_i, z_i; \theta_i)} = \frac{v'(z_i/\theta_i)}{\gamma\theta_i}, \qquad i \in \mathcal{I}. \tag{1.5}$$

Three points are worth noting in the gross-income/consumption space. First, i's indifference curves are parallel *vertical* displacements of each other. Second, the *Spence-Mirrlees condition* is met : for a given gross income level, the higher the productivity of an individual, the flatter his indifference curves. Third, when the marginal utility of money γ increases, indifference curves become flatter ; so a lower increase in consumption is required to compensate for an increase in gross income while keeping utility constant.

A social allocation specifies a consumption and gross income level for each individual. It is represented by a vector $a = (x, z) \in \mathbb{R}^I \times \mathbb{R}^I_+$, with $x = (x_1, \ldots, x_I)$ and $z = (z_1, \ldots, z_I)$. The tax policymaker knows the functional form of the utility function and the distribution of wages in the population. He is however unable to observe each individual's productivity. As a result, he is restricted to setting taxes as a function of gross income z_i. By the taxation principle, a non-linear income tax schedule is therefore a mapping

$$\begin{cases} \theta & \longrightarrow \quad a \qquad \mathbb{R} \times \mathbb{R}_+ \\ \theta_i & \qquad\qquad (x_i, z_i), \end{cases} \tag{1.6}$$

which satisfies the *incentive compatibility constraints*

$$IC_{ij} : u(x_i, z_i; \theta_i) \geq u(x_j, z_j; \theta_i), \qquad \forall (i, j) \in \mathcal{I}^2, \tag{1.7}$$

and the *tax revenue constraint*

$$\sum_{i=1}^{I} z_i \geq \sum_{i=1}^{I} x_i. \tag{1.8}$$

An allocation a is *production efficient* if the budget-balanced constraint (1.8) is binding.

The *social welfare function* $W : \mathbb{R}^I \times \mathbb{R}^I_+ \to \mathbb{R}$ is a weighted sum of individual utilities,

$$W(a) := \sum_{i=1}^{I} \lambda_i u(x_i, z_i; \theta_i), \tag{1.9}$$

in which $\lambda := (\lambda_1, \ldots, \lambda_I)$ are individual social weights. The policymaker's taste for redistribution from the high to the low productive individuals is captured through the requirement that the higher the individual productivity the less the weight in the social objective, i.e.

$$0 < \lambda_I < \ldots < \lambda_1. \tag{1.10}$$

If $I = 2$, this assumption amounts to considering the "normal" case studied by Stiglitz (1982) in which only the incentive compatibility constraint of the high type is binding. By extension, when $I > 2$, one can therefore expect that the only binding incentive compatibility constraints will be the downward adjacent ones stating that $i + 1$ must be indifferent between his own bundle and i's one, for $i = 1, \ldots, I - 1$.

As $W(a)$ is homogeneous of degree one in λ, the sum of the social weights can be normalized without loss of generality. It is convenient to define $\Lambda(\theta_i)$ as the cumulative social weight of the i less productive individuals, and to set

$$\Lambda(\theta_I) = I. \tag{1.11}$$

Consequently, *admissible parameters* $(\theta, \gamma, \lambda)$ belong to the set

$$\mathcal{P} := \{\theta | \, (1.1) \text{ is satisfied}\} \times \mathbb{R}_{++} \times \{\lambda | (1.10) \text{ and } (1.11) \text{ are satisfied}\}. \tag{1.12}$$

The optimal non-linear income tax problem can thus be formulated as follows.

Problem 1.1 (Optimal Non-linear Income Tax Problem) *For $(\theta, \gamma, \lambda) \in \mathcal{P}$, choose an allocation $a \in \mathbb{R}^I \times \mathbb{R}^I_+$ to maximize $W(a)$ under the incentive-compatibility constraints (1.7) and the tax revenue constraint (1.8).*

1.3. THE OPTIMAL ALLOCATION

The optimal non-linear income tax problem involves two sets of control variables, gross income z and net income x. It can however be transformed into a reduced-form problem in which the policymaker chooses only one of these sets. The reduced-form problem makes it easier to interpret the social value function as well as the optimality conditions and to derive comparative static results. For this purpose, Problem 1.1 is separated into two subproblems. In the first one, gross income is arbitrarily chosen within the set of incentive-feasible gross income levels $\mathcal{Z}$ (which will be formally defined below).

Subproblem 1.1 *Given a gross income vector $z \in \mathcal{Z}$ and the parameters $(\theta, \gamma, \lambda) \in \mathcal{P}$, choose the consumption vector $x \in \mathbb{R}^I$ to maximize the social welfare function $W(a)$ subject to the incentive-compatibility constraints (1.7) and the tax revenue constraint (1.8).*

Let $\mathcal{X}^*(z;\theta,\gamma,\lambda)$ be the set of maximizers. Then, if there is a unique consumption vector $x^*(z;\theta,\gamma,\lambda)$ in $\mathcal{X}^*(z;\theta,\gamma,\lambda)$, the solution in z to Problem 1.1 is obtained as

$$\arg\max_{z\in\mathcal{Z}} W\left(x^*\left(z;\theta,\gamma,\lambda\right),z\right). \tag{1.13}$$

So, the reduced-form problem can be stated as follows.

Subproblem 1.2 *Given the parameters $(\theta,\gamma,\lambda)\in\mathcal{P}$, choose $z\in\mathcal{Z}$ to maximize the social welfare function $W\left(x^*\left(z;\theta,\gamma,\lambda\right),z\right)$.*

1.3.1. Implications of the Incentive-Compatibility Constraints

The incentive-compatibility constraints (1.7) place structure on the solution to Problem 1.1. These restrictions can be used to derive sufficient conditions under which an allocation a is incentive-compatible. We proceed in two steps.

First, if an allocation a satisfies (1.7), then the two adjacent incentive-compatibility constraints $IC_{i+1,i}$ and $IC_{i,i+1}$ are satisfied for $i=1,\ldots,I-1$. Adding both of them yields

$$R\left(z_i;\theta_i,\theta_{i+1}\right)\leq R\left(z_{i+1};\theta_i,\theta_{i+1}\right),\qquad i=1,\ldots,I-1. \tag{1.14}$$

where the function $R:\mathbb{R}_+\to\mathbb{R}$ is defined by

$$R\left(z_k;\theta_i,\theta_j\right):=v\left(z_k/\theta_i\right)-v\left(z_k/\theta_j\right),\qquad (k,i,j)\in\mathcal{I}^3. \tag{1.15}$$

Lemma 1.1 *Let $(\theta_i,\theta_j)\in\mathbb{R}^2_{++}$. Then, $R\left(z_k;\theta_i,\theta_j\right)$ is strictly increasing and strictly convex in z_k if and only if $\theta_i>\theta_j$.*

Proof. See the Appendix. ■

For $i=1,\ldots,I-1$, applying Lemma 1.1 to (1.14) yields $z_{i+1}\geq z_i$. So, since $v'>0$, $IC_{i+1,i}\Leftrightarrow x_{i+1}-x_i\geq v\left(z_{i+1}/\theta_{i+1}\right)-v\left(z_i/\theta_{i+1}\right)$ implies $x_{i+1}\geq x_i$. Consequently, the gross income and net income vectors of an incentive-compatible allocation a must be non-decreasing in productivity, i.e. such that

$$\left(x_1,z_1\right)\leq\cdots\leq\left(x_I,z_I\right), \tag{1.16}$$

with $\left(x_{i-1},z_{i-1}\right)<<\left(x_i,z_i\right)$ if $\left(x_{i-1},z_{i-1}\right)\neq\left(x_i,z_i\right)$, $i=2,\ldots,I$. Accordingly, the set $\mathcal{Z}$ in which the solution in z to Problem 1.1 must lie is defined as

$$\mathcal{Z}:=\left\{z\in\mathbb{R}^I|0\leq z_1\leq\cdots\leq z_I\right\}. \tag{1.17}$$

The condition $z\in\mathcal{Z}$ corresponds to the second-order condition for incentive compatibility in the continuum model.

Second, given $z\in\mathcal{Z}$, it proves sufficient to check the downward and upward adjacent incentive compatibility constraints to get an incentive-compatible allocation provided $z\in\mathcal{Z}$ (Cooper,

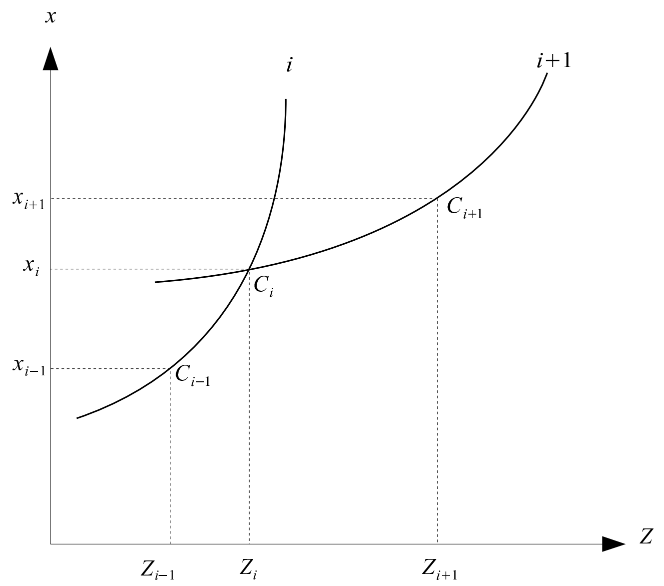

FIG. 1.1 – Incentive Compatibility of a Simple Monotonic Chain to the Left

1984).

Lemma 1.2 *Given z in $\mathcal{Z}$,*

$$u(x_i, z_i; \theta_i) \geq u(x_{i-1}, z_{i-1}; \theta_i), \qquad i = 2, \ldots, I \tag{1.18}$$

$$u(x_i, z_i; \theta_i) \geq u(x_{i+1}, z_{i+1}; \theta_i), \qquad i = 1, \ldots, I-1, \tag{1.19}$$

imply (1.7).

Proof. See the Appendix. ■

There are many ways of satisfying (1.18). This is notably the case if all adjacent downward incentive-compatibility constraints are binding, i.e. if

$$u(x_{i+1}, z_{i+1}; \theta_{i+1}) = u(x_i, z_i; \theta_{i+1}), \qquad i = 1, \ldots, I-1. \tag{1.20}$$

An allocation satisfying (1.20) is called a *simple monotonic chain to the left* by Guesnerie and Seade (1982).

Proposition 1.1 *Let an allocation $a \in \mathbb{R}^I \times \mathbb{R}_+^I$ be a simple monotonic chain to the left and $z \in \mathcal{Z}$. Then a satisfies the incentive compatibility constraints (1.7).*

This result is established geometrically in Figure 1.1. Consider three successive gross income/consumption bundles $\mathcal{C}_{i-1} \leqq \mathcal{C}_i \leqq \mathcal{C}_{i+1}$ and suppose person k is indifferent between $\mathcal{C}_k$

and $\mathcal{C}_{k-1}$ for $k = i, i+1$, i.e. $u(\mathcal{C}_i;\theta_i) = u(\mathcal{C}_{i-1};\theta_i)$ and $u(\mathcal{C}_i;\theta_{i+1}) = u(\mathcal{C}_{i+1};\theta_{i+1})$. Since the Spence-Mirrlees condition is met, i (weakly) prefers $\mathcal{C}_i$ to $\mathcal{C}_{i+1}$. As a consequence, the local upward incentive-compatibility constraint is necessarily satisfied. By Lemma 1.2, all non-adjacent incentive-compatibility constraints for i are also verified. Repeating the argument for $i = 2, \ldots, I-1$, shows that any simple monotonic chain to the left is incentive-compatible.

A simple-monotonic chain to the left reflects a specific efficiency/rent-extraction trade-off. Indeed, given quasilinear-in-consumption preferences, (1.20) is equivalent to

$$u(x_{i+1}, z_{i+1}; \theta_{i+1}) - u(x_i, z_i; \theta_i) = R(z_i; \theta_i, \theta_{i+1}), \qquad i = 1, \ldots, I-1. \tag{1.21}$$

By Lemma 1.1, (1.21) indicates at which rate utility must be convexly increased for the tax schedule to induce individual truthtelling. So, for each pair of adjacent productivity levels (θ_i, θ_{i+1}), $R(z_i; \theta_i, \theta_{i+1})$ may be regarded as the *marginal rent* the policymaker has to leave to the more productive $i+1$-individual because of the informational externality. Consequently, (1.21) constitutes the discrete analogue of the first-order condition for incentive compatibility obtained in the models with a continuum of individuals.

1.3.2. Optimal Consumption Given Fixed Levels of Income

Since $\mathcal{Z}$ is closed and bounded whilst $W(a)$ is continuous, there exist solutions to Subproblem 1.1 for all gross income vector $z \in \mathcal{Z}$ and all $(\theta, \gamma, \lambda) \in \mathcal{P}$. They all share the following remarkable structure.

Lemma 1.3 *Given $z \in \mathcal{Z}$ and $(\theta, \gamma, \lambda) \in \mathcal{P}$, any allocation $a = (x^*, z)$ where $x^* \in \mathcal{X}^*(z; \theta, \gamma, \lambda)$, is a* simple monotonic chain to the left *which is* production efficient*.*

Proof. See the Appendix. ■

Combined with Proposition 1.1, Lemma 1.3 ensures that all implications of the incentive-compatibility constraints (1.7) are embedded in any solution to Subproblem 1.1, provided $z \in \mathcal{Z}$. Moreover, the fact that a is a simple monotonic chain to the left gives rise to a specific consumption pattern. Indeed, by (1.20),

$$x_i = x_{i-1} + \frac{1}{\gamma}\left[v(z_i/\theta_i) - v(z_{i-1}/\theta_i)\right], \qquad i = 2, \ldots, I, \tag{1.22}$$

and so

$$x_i = x_1 + \frac{1}{\gamma}\sum_{j=2}^{i}\left[v(z_j/\theta_j) - v(z_{j-1}/\theta_j)\right], \qquad i = 2, \ldots, I. \tag{1.23}$$

As any solution to Subproblem 1.1 is production efficient, by Lemma 1.3, the binding tax revenue constraint (1.8) can be substituted in $\sum_{i=1}^{I} x_i$, obtained from (1.23), to get

$$\sum_{i=1}^{I} z_i = Ix_1 + \frac{1}{\gamma}\sum_{i=2}^{I}\sum_{j=2}^{i}\left[v\left(\frac{z_j}{\theta_j}\right) - v\left(\frac{z_{j-1}}{\theta_j}\right)\right] = Ix_1 + \frac{1}{\gamma}\sum_{i=2}^{I}(I+1-i)\left[v\left(\frac{z_i}{\theta_i}\right) - v\left(\frac{z_{i-1}}{\theta_i}\right)\right]. \tag{1.24}$$

This equation admits a unique solution in x_1. Substituting the latter in (1.23) and proceeding sequentially show that there is a unique consumption vector in $\mathcal{X}^*(z;\theta,\gamma,\lambda)$, which is independent of the social weights λ and inherits the differentiability properties of v.

Proposition 1.2 *Given $z \in \mathcal{Z}$ and $(\theta,\gamma,\lambda) \in \mathcal{P}$, the* unique *function solution to Subproblem 1.1 is twice continuously differentiable, defined by $x^* : \mathcal{Z} \times \mathbb{R}^I_{++} \times \mathbb{R}_{++} \to \mathbb{R}^I_{+}$ with*

$$x_1^*(z;\theta,\gamma) = \frac{1}{I}\left\{\sum_{j=1}^{I} z_j - \frac{1}{\gamma}\sum_{j=2}^{I}(I+1-j)\left[v\left(\frac{z_j}{\theta_j}\right) - v\left(\frac{z_{j-1}}{\theta_j}\right)\right]\right\}, \tag{1.25}$$

$$x_i^*(z;\theta,\gamma) = x_1^*(z;\theta,\gamma) + \frac{1}{\gamma}\sum_{j=2}^{i}\left[v\left(\frac{z_j}{\theta_j}\right) - v\left(\frac{z_{j-1}}{\theta_j}\right)\right], \qquad i = 2,\ldots,I. \tag{1.26}$$

1.3.3. The Reduced Form

We can now take stock of the previous results to give a more compact formulation of Subproblem 1.2. For this purpose, it is convenient to introduce the new vector of social parameters $\beta = (\beta_1,\ldots,\beta_I)$ with

$$\beta_i := \Lambda(\theta_i) - i, \qquad i \in \mathcal{I}. \tag{1.27}$$

Because of (1.10) and (1.11), the graph of $i \to \Lambda(\theta_i)$ is hump-shaped and above the 45°-line. Hence, $\beta_i > 0$ for $i = 1,\ldots,I-1$, whilst $\beta_I = 0$. Skill levels do not appear directly in (1.27), unlike in the analogous expression in Weymark (1986b).

The parameters β_i summarize in a transparent way the redistributive taste of the government. First, they would all be equal if the government adopted pure utilitarianism as a social objective, in which case $\beta_i = 0$ for every i. Therefore, the social parameters β_i express the policymaker's *strict* aversion to income inequality. Second, to get further insight into β_i, it is instructive to consider the effects of the government's decision to give each of the i less productive individuals one extra euro of consumption in the absence of incentive-compatibility constraints. On the one hand, the utility of each of them is increased by γ, the marginal utility of money. Hence, the gross social benefit amounts to $\gamma\Lambda(\theta_i)$. On the other hand, the consumption of every individual $i = 1,\ldots,I$ must be decreased by i/I in order to satisfy the tax revenue constraint (1.8). This reduces individual welfare by $\gamma i/I$ and thus social welfare by $\Lambda(\theta_I) \times \gamma i/I = \gamma i$. Summing both effects, it appears that $\gamma\beta_i$ is the net social benefit of marginally increasing the consumption of the i less skilled individuals whilst ignoring informational externalities. So, β_i is this net social benefit expressed in monetary units. Third, the parameters β_i can alternatively be defined as

$$\beta_i := I - i - \sum_{j=i+1}^{I} \lambda_j, \qquad i = 1, \ldots, I-1. \tag{1.28}$$

They thus also corresponds to the net social cost, expressed in euros, of marginally increasing the consumption of the $I - i$ most productive individuals. That is why they are henceforth referred to as *net cumulative social weights*. Thanks to them Subproblem 1.2 can be rewritten as follows.

Problem 1.2 (Reduced Form) *For $(\theta, \gamma, \lambda) \in \mathcal{P}$, choose $z \in \mathcal{Z}$ so as to maximize the social objective function $\mathcal{W}^* (z; \theta, \gamma, \lambda) : \mathcal{Z} \times \mathcal{P} \to \mathbb{R}$ with*[4]

$$\mathcal{W}^* (z; \theta, \gamma, \lambda) := \sum_{i=1}^{I} \left[\gamma z_i - v (z_i/\theta_i)\right] - \sum_{i=1}^{I} \beta_i R (z_i; \theta_i, \theta_{i+1}). \tag{1.29}$$

$\mathcal{W}^*$ is strictly concave over the convex set $\mathcal{Z}$ because v'' is positive and increasing.[5] Hence, there is a *unique* gross income vector which maximizes $\mathcal{W}^*$ for every $(\theta, \gamma, \lambda) \in \mathcal{P}$. It remains to substitute it in $x_i^* (g^z (\theta, \gamma, \lambda); \theta, \gamma)$ to get the optimal allocation.

Proposition 1.3 *For $(\theta, \gamma, \lambda) \in \mathcal{P}$, there is a unique allocation $a = (g^x (\theta, \gamma, \lambda), g^z (\theta, \gamma, \lambda))$ solution to Problem 1.2, with*

$$\begin{cases} g_i^z (\theta, \gamma, \lambda) = \arg\max_{z_i \in \mathcal{Z}} \mathcal{W}^* (z; \theta, \gamma, \lambda) \\ g_i^x (\theta, \gamma, \lambda) = x_i^* (g^z (\theta, \gamma, \lambda); \theta, \gamma) \end{cases}, \qquad i \in \mathcal{I}. \tag{1.30}$$

Proof. See the Appendix. ■

A first observation is that both $g^x (\theta, \gamma, \lambda)$ and $g^z (\theta, \gamma, \lambda)$ are functions. Moreover, Proposition 1.3 implies that the social allocation solution to the optimal non-linear income tax problem is a monotonic chain to the left. As a consequence, the optimal tax schedule is not differentiable at each observed gross income level z_i. It is nevertheless possible to use the differentiability of the indifference curves in order to define implicit marginal tax rates as

$$T' (z_i; \theta_j) := 1 - s (z_i; \theta_j) = 1 - \frac{v' (z_i/\theta_j)}{\gamma \theta_j}, \qquad (i, j) \in \mathcal{I}^2. \tag{1.31}$$

Every $T' (z_i; \theta_j)$ is less than one since $v' > 0$ and decreases in z_i since $v'' > 0$. Since at the optimum only the adjacent downward incentive-compatibility constraints are binding, two implicit marginal tax rates are of particular interest at each observed gross income level z_i : the implicit marginal tax rate $T' (z_i; \theta_i)$ faced by i for whom the $(x_i^* (z_i; \theta, \gamma), z_i)$-bundle is designed and the

[4] Since $\beta_I = 0$, $R (z_I; \theta_I, \theta_{I+1})$ is defined arbitrarily.

[5] Since $v'' > 0$, $\beta_i \geq 0$ and Lemma 1.1 holds, $d^2 \mathcal{W}^* (z; \theta, \gamma, \lambda) / dz_i^2 = -v'' \left(\frac{z_i}{\theta_i}\right) / \theta_i^2 - \beta_i R'' (z_i; \theta_i, \theta_{i+1})$ is strictly negative.

implicit marginal tax rate $T'(z_i;\theta_{i+1})$ the nearest more productive $i+1$-individual would face if he were mimicking the i-individual.

The implicit marginal tax rates allow us to get further understanding of the reduced-form objective function $\mathcal{W}^*(z;\theta,\gamma,\lambda)$. Indeed, let z be a fixed gross income vector and consider that the gross income z_i of the θ_i-individual is increased at the margin. As

$$\frac{d\mathcal{W}^*(z;\theta,\gamma,\lambda)}{dz_i} = \gamma T'(z_i;\theta_i) - \beta_i R'(z_i;\theta_i,\theta_{i+1}), \qquad i \in \mathcal{I}, \tag{1.32}$$

by (1.29) and (1.31), the impact on social welfare may be thought of as proceeding in two steps. In the first step, the θ_i-individual pays $T'(z_i;\theta_i)$ additional euros in taxes, which relaxes the tax revenue constraint (1.8). As γ is the marginal utility of money, the positive effect on social welfare amounts to $\gamma T'(z_i;\theta_i)$. In the second step, the effect on incentives is taken into account. Person i receives $1-T'(z_i;\theta_i)$ extra euro of consumption. As a result, the $I-i$ more productive individuals have to sacrifice less consumption when they decide to mimic i. So, cheating becomes more attractive to them. For $i+1$, the marginal rent is $R'(z_i;\theta_i,\theta_{i+1})$. This person's binding incentive-compatibility constraint $IC_{i+1,i}$ is restored by adjusting consumption by $R'(z_i;\theta_i,\theta_{i+1})/\gamma$. Increasing the consumption of everybody of higher ability by the same amount preserves the monotonic chain to the left. Because $\gamma\beta_i$ is the net social cost of marginally increasing consumption of the $I-i$ most productive individuals, social welfare is reduced by $\beta_i R'(z_i;\theta_i,\theta_{i+1})$. If the social optimum is interior, it is therefore obtained when the positive effect on social welfare due to the relaxation of the tax revenue constraint offsets the negative one stemming from private information, i.e.

$$\frac{d\mathcal{W}^*(z;\theta,\gamma,\lambda)}{dz_i} = 0 \Leftrightarrow T'(z_i;\theta_i) = \frac{\beta_i}{\gamma} R'(z_i;\theta_i,\theta_{i+1}), \qquad i \in \mathcal{I}. \tag{1.33}$$

1.3.4. Characterization of the Optimal Allocation

In order to characterize the optimal allocation, it is useful to consider the *relaxed* form of Problem 1.2, obtained when all monotonicity conditions on z are removed (but not the non-negativity constraint $z \geqq 0$). Its unique solution is denoted $\widehat{z}(\theta,\gamma,p) = (\widehat{z}_1(\theta,\gamma,p),\ldots,\widehat{z}_I(\theta,\gamma,p))$. If $\widehat{z}(\theta,\gamma,p)$ is non-decreasing, then it is equal to the socially optimal gross income vector $g^z(\theta,\gamma,\lambda)$. Otherwise, the optimal allocation involves bunching.

Proposition 1.4 *For $(\theta,\gamma,\lambda) \in \mathcal{P}$, the optimal allocation is such that :*
(i) $g_i^z(\theta,\gamma,\lambda) = \widehat{z}_i(\theta,\gamma,p)$, except on a finite number K of disjoint compact sets $\mathcal{B}^k := \{i^k,\ldots,j^k\}$, $k=1,\ldots,K$, i^k increasing with k, where $g_n^z(\theta,\gamma,\lambda) = \overline{z}^k$ for every $n \in \mathcal{B}^k$;
(ii) $0 < T'(z_i;\theta_i) < 1$ for $i=1,\ldots,I-1$, $T'(z_I;\theta_I)=0$ and bunching at the top is ruled out;
(iii) if $i^k = 1$,

$$\frac{1}{\#\mathcal{B}^k}\sum_{n\in\mathcal{B}^k} \left.\frac{\partial\mathcal{W}^*(z;\theta,\gamma,\lambda)}{\partial z_n}\right|_{z_n=\overline{z}^k} \leq 0 \;\left(=0 \text{ if } \overline{z}^k > 0\right); \tag{1.34}$$

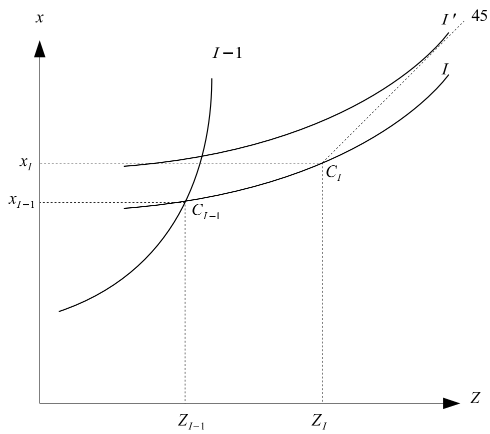

FIG. 1.2 – No Distortion at the Top

(iv) for each interior $\mathcal{B}^k$,

$$\frac{1}{\#\mathcal{B}^k} \sum_{n \in \mathcal{B}^k} \left. \frac{\partial \mathcal{W}^* (z; \theta, \gamma, \lambda)}{\partial z_n} \right|_{z_n = \overline{z}^k} = 0. \tag{1.35}$$

Proof. See the Appendix. ■

The optimal gross income vector is thus solution to the relaxed Problem 1.2, except on a finite number of bunching sets. Two types of bunching must be distinguished. Bunching due to the violation of the non-negativity constraints, called $z = 0$ bunching by Boone and Bovenberg (2007), can only occur at the bottom, in which case $\overline{z}^1 = 0$. The other kind of bunching stems from the violation of the monotonicity constraints and can happen either at the bottom or in the interior of the skill distribution. In this situation, the gross income level $\overline{z}^k$ of the individuals who are bunched together is implicitly determined by the average of their first-order conditions for the relaxed Problem 1.2. The fact that there is no bunching and non distortion at the top follows from an efficiency argument, illustrated in Figure 1.2. To see why, assume $T'(z_I; \theta_I) \neq 0$ and consider the 45° line through (x_I, z_I). Moving along this line above (x_I, z_I) increases I's utility without hurting neither the tax revenue constraint (1.8) nor the incentive-compatibility constraints (1.7) since a new simple monotonic chain to the left can easily be constructed. Therefore, the initial situation is strongly Pareto-dominated and $T'(z_I; \theta_I) = 0$ at the optimum. Now, assume that $I - 1$ and I are bunched together (possibly with other individuals). They both face a marginal tax rate equal to zero. Hence,

$$T'(z_I; \theta_I) = 0 \Leftrightarrow z_I = \theta_I v'^{-1}(\gamma \theta_I), \tag{1.36}$$

$$T'(z_{I-1}; \theta_{I-1}) = 0 \Leftrightarrow z_{I-1} = \theta_{I-1} v'^{-1}(\gamma \theta_{I-1}). \tag{1.37}$$

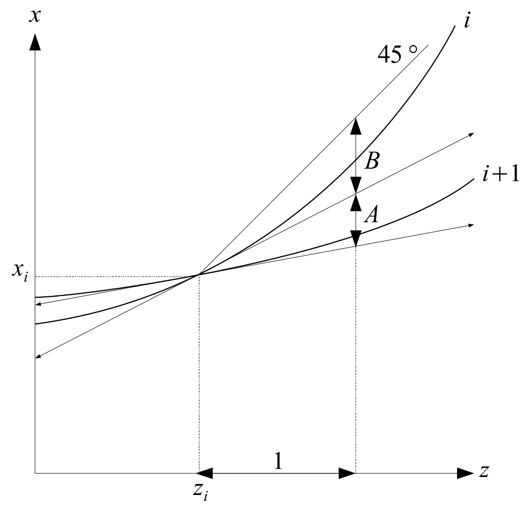

FIG. 1.3 – $\alpha_i\left(z_i;\theta_i,\theta_{i+1},\gamma\right)=A/B$

Since they are bunched together, it must be $z_I=z_{I-1}$. However, this is impossible because $v''>0$ implies that $\theta v'^{-1}(\gamma\theta)$ is strictly increasing in θ.

To gain further insights into the optimal allocation, it is useful to define $\alpha_i:[0,\theta_i v'^{-1}(\gamma\theta_i)[\to\mathbb{R}_+$ as

$$\alpha_i\left(z_i;\theta_i,\theta_{i+1},\gamma\right):=\frac{R'\left(z_i;\theta_i,\theta_{i+1}\right)}{\gamma T'\left(z_i;\theta_i\right)}=\frac{T'\left(z_i;\theta_{i+1}\right)-T'\left(z_i;\theta_i\right)}{T'\left(z_i;\theta_i\right)},\qquad i=1,\ldots,I-1. \tag{1.38}$$

For $i<I$, the domain of α_i corresponds to the gross incomes for which $T'\left(z_i;\theta_i\right)$ is strictly positive ; hence, by Proposition 1.4, $g_i^z(\theta,\gamma,\lambda)\in[0,\theta_i v'^{-1}(\gamma\theta_i)[$. Moreover, α_i is continuous and strictly increasing over its domain.[6] Using (1.33) and (1.38), $\widehat{z}$ must satisfy

$$\alpha_i\left(\widehat{z}_i;\theta_i,\theta_{i+1},\gamma\right)\geq\frac{1}{\beta_i}\ \ (=\ \text{if}\ \widehat{z}_i>0)\,,\qquad i=1,\ldots,I-1. \tag{1.39}$$

So, if $\widehat{z}_i>0$, $\widehat{z}_i=\alpha_i^{-1}\left(1/\beta_i;\theta_i,\theta_{i+1},\gamma\right)$. Since the optimal gross income vector $g^z(\theta,\gamma,\lambda)$ lies in the interior of $\mathcal{Z}$ if and only if $0<\widehat{z}_1<\ldots<\widehat{z}_I$, the following characterization is obtained.

Proposition 1.5 *For $(\theta,\gamma,\lambda)\in\mathcal{P}$, the optimum is fully separating if and only if $\beta_i\in\alpha_i\left([0,\theta_i v'^{-1}(\gamma\theta_i)[\right)$ for $i=1,\ldots,I-1$ with*

$$0<\alpha_1^{-1}\left(1/\beta_1;\theta_1,\theta_2,\gamma\right)<\ldots<\alpha_{I-1}^{-1}\left(1/\beta_{I-1};\theta_{I-1},\theta_I,\gamma\right)<\theta_I v'^{-1}\left(\gamma\theta_I\right). \tag{1.40}$$

[6] For $i<I$, α_i is strictly increasing since $dT'\left(z_i;\theta_i\right)/dz_i<0$ and $R''\left(z_i;\theta_i,\theta_{i+1}\right)>0$ by Lemma 1.1.

This proposition implicitly characterizes the set of parameters for which there is no bunching, which is denoted $\mathcal{P}^0$ for easy reference.[7] When $(\theta, \gamma, \lambda) \in \mathcal{P}^0$, if follows from (1.39) that the optimality conditions can be written in a strikingly simple form.

Proposition 1.6 *For $(\theta, \gamma, \lambda) \in \mathcal{P}^0$, z is socially optimal if and only if*

$$\alpha_i (z_i; \theta_i, \theta_{i+1}, \gamma) = 1/\beta_i, \qquad i = 1, \ldots, I-1, \tag{1.41}$$

and $T'(z_I; \theta_I) = 0$.

For a gross income z_i, $\alpha_i (z_i; \theta_i, \theta_{i+1}, \gamma)$ tells us to to which extent $i+1$ must face a higher implicit marginal tax rate than i. Geometrically, it thus corresponds to the tangent of the angle between the indifference curves of i and $i+1$ divided by $T'(z_i; \theta_i)$, as shown in Figure 1.3. The wedge $\alpha_i (z_i; \theta_i, \theta_{i+1}, \gamma)$ is closely related to the single-crossing condition and thus henceforth referred to as the *Spence-Mirrlees wedge.* Indeed, the single-crossing condition corresponds to a restriction on its *sign*, which must be strictly positive. This condition is herein automatically satisfied because individual preferences are quasilinear. The conditions for social optimality (1.41) introduce an additional restriction on the *magnitude* of $\alpha_i (z_i; \theta_i, \theta_{i+1}, \gamma)$: an allocation is socially optimal only if, at each observed gross income level z_i, the Spence-Mirrlees wedge is entirely determined by the exogenously given cumulative social weight β_i. In addition, efficiency requires the labour supply of the more productive individuals not to be distorted. Hence, by (1.5), one obtains $z_I = \theta_I v'^{-1}(\gamma\theta_I)$, which is independent of the social weights.

In the absence of bunching, Proposition 1.6 allows a simple two-step geometric construction of the optimal allocation, as illustrated in Figure 1.4. *In the first step*, the tax revenue constraint is ignored. Start at $(0, \widehat{x}_1)$, where $\widehat{x}_1$ is arbitrary. Move along θ_1's indifference curve until $\alpha_1 (z_1; \theta_1, \theta_2, \gamma) = 1/\beta_1$. This point is $(\overline{x}_1, z_1)$. Then, move along θ_2's indifference curve through $(\overline{x}_1, z_1)$ until $\alpha_2 (z_2; \theta_2, \theta_3, \gamma) = 1/\beta_2$. This point is $(\overline{x}_1, z_1)$. And so on until $i = I-1$. The determination of z_I exploits the no-distortion-at-the-top result. Starting from $(\overline{x}_{I-1}, z_{I-1})$, gross income is increased along the indifference curve of the most productive individual until the tangent to this line has slope one. By construction, the allocation $(\overline{x}, z)$ is a simple monotonic chain to the left and is thus incentive compatible. However, it is not necessarily budget-balanced. That is why, *in the second step*, each $\overline{x}_i$ is varied by a same amount $\varepsilon = \frac{1}{I}\sum_{i=1}^{I}(z_i - \overline{x}_i)$ so as to get a binding tax revenue constraint. The resulting incentive-compatible and production-efficient allocation $(x, z) = (\overline{x} + \varepsilon, z)$ is socially optimal.

Before going further and derive comparative static properties, it is instructive to examine one main source of differences between our results and those derived in Weymark (1987, 1986b,a). In the latter papers, the quasilinear-in-leisure utility function $u(x_i, z_i; \theta_i) := h(x_i) - \gamma z_i/\theta_i$ is replaced by its monotone transform $\widetilde{u}(x_i, z_i; \theta_i) = \theta_i h(x_i) - \gamma z_i$ in order to replace $\sum_{i=1}^{I} z_i$ by

[7] α_i is type-dependent. Therefore, contrary to Weymark (1986a), condition (1.40) does not only depend on β, but also on θ and γ.

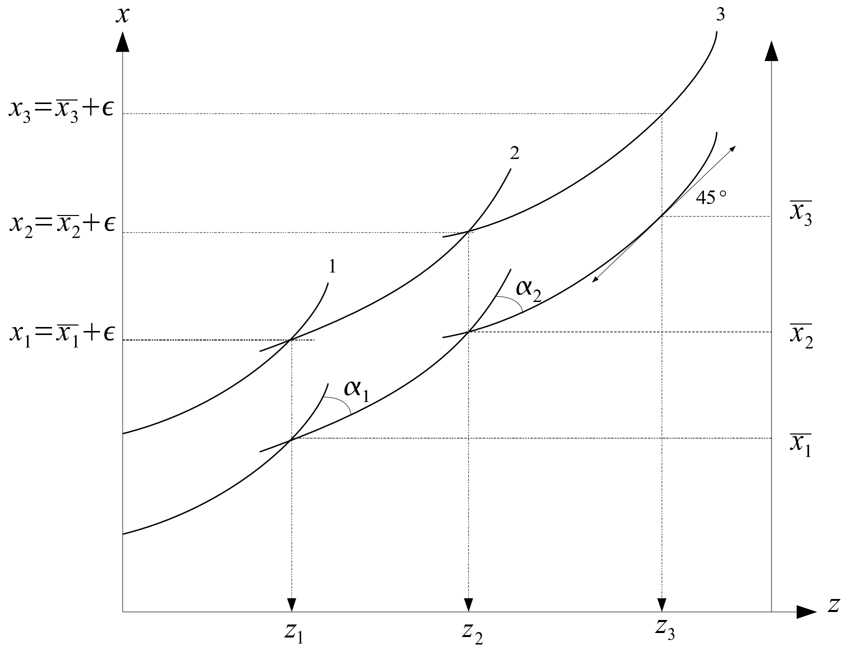

FIG. 1.4 – Construction of the Optimal Allocation

$\sum_{i=1}^{I} x_i$ and get

$$\sum_{i=1}^{I} \widetilde{u}\left(x_i, z_i; \theta_i\right) = \sum_{i=1}^{I} \theta_i h\left(x_i\right) - \gamma \sum_{i=1}^{I} z_i = \sum_{i=1}^{I} \theta_i h\left(x_i\right) - \gamma \sum_{i=1}^{I} x_i.^{8} \tag{1.42}$$

This step is required to obtain a reduced-form optimal income tax problem. This is why skill-normalized social weights $\widetilde{\lambda}_i := \lambda_i/\theta_i$ are used in the social objective $\sum_{i=1}^{I} \lambda_i u\left(x_i, z_i; \theta_i\right) = \sum_{i=1}^{I} \widetilde{\lambda}_i \widetilde{u}\left(x_i, z_i; \theta_i\right)$. The first-order conditions of the reduced-form problem involve therefore skilled-normalized cumulative social weights $\sum_{j=1}^{i} \widetilde{\lambda}_j$ instead of Λ_i. So, the impact of the policy-maker's taste for redistribution is less transparent because social weights and productivity levels are mixed together.

In Weymark's reduced-form problem, the social objective is maximized with respect to consumption levels. The parameters $\sum_{j=1}^{i} \widetilde{\lambda}_j$, θ_i and θ_{i+1} appear in the first-order condition for x_i. As a consequence, x_i is a function of all social weights λ_j and *all* productivity levels. A contrario, in the present setting, the variable with respect to which the reduced-form objective is maximized, z_i, does not depend on θ_k for $k \neq i, i+1$. Therefore, one expects the comparative statics to differ significantly, from one kind of quasilinearity to the other, when skill levels are directly involved in the analysis.

[8] Cf. (A.13) in Weymark (1986b).

1.4. COMPARATIVE STATIC PROPERTIES

Besides providing a geometric interpretation of the optimality conditions, the reduced form makes it possible to derive comparative static results of the optimal income tax allocation. For this purpose, it is first necessary to examine the differentiability properties of the main functions $g^x(\theta, \gamma, \lambda)$ and $g^z(\theta, \gamma, \lambda)$. Using Property 1.4 is is possible to establish that both functions are continuously differentiable provided a change in $(\theta, \gamma, \lambda)$ does not modify the sets $\mathcal{B}^k$ of individuals bunched together. In this case, calculus techniques makes it possible to obtain comparative static properties. However, for expositional reasons, the analysis will henceforth consider that the initial allocation at stake is fully separating.

Proposition 1.7 *The functions g^x and g^z are $\mathcal{C}^1$ at every $(\theta, \gamma, \lambda)$ in $\mathcal{P}^0$.*

Proof. See the Appendix. ■

1.4.1. Comparative Statics for the Marginal Utility of Money

The marginal utility of money γ measures the intensity of the individual preference for private consumption. When it goes up, the marginal utility of leisure expressed in consumption good is reduced. So, the indifference curves of every individual become flatter in the (z, x)-space. Everyone is thus willing to sacrifice more leisure to obtain a certain amount of additional consumption. From a more general viewpoint, changing γ at the margin casts light on how taxes should be adjusted in a country where all individuals would like to work more to consume more.

Proposition 1.8 *For $(\theta, \gamma, \lambda) \in \mathcal{P}^0$ and $i \in \mathcal{I}$, $\partial g_i^z(\theta, \gamma, \lambda) / \partial\gamma > 0$.*

Proof. See the Appendix. ■

The changes in the tax system should not discourage more hard-working individuals to earn more. National income is increased and thus, since production efficiency is preserved by Lemma 1.3, total consumption goes up. However, as shown by Proposition 1.2, the change in person i's consumption depends on how the gaps $v(z_j/\theta_j) - v(z_{j-1}/\theta_j)$, $j = 1, \ldots, i$, react to a change in the marginal utility of money. Because there is no reason why these gaps should be affected in a systematic manner, the effect of varying γ on consumption cannot be signed without introducing further restrictions. In the same vein, the changes in the implicit marginal tax rates $T(z_i; \theta_i)$ cannot be obtained since z_i and γ are simultaneously increased.

1.4.2. Comparative Statics for Individual Productivities

In Mirrlees's model, individuals are born with different abilities to turn effort into output. These fixed skill levels are the sole source of heterogeneity within the population. They probably constitute the most basic ingredients of the model as they give rise to the adverse selection problem. Indeed, the effort level z_j/θ_i a given i-individual must provide to earn the gross income of everyone else depends on his productivity. In practice, skill levels can be subject to changes. For instance, a high skilled individual can catch an illness which impairs his productivity while a low skilled can benefit from on-the-job training. Another argument rests on the technological side of the economy. Since a person's productivity depends on the capitalistic intensity in his branch of activity, a new investment can make him more productive. It seems therefore worthwhile to examine what is the impact of changing a person's skill level on his own choices, but also on the policy-maker and the whole population.[9]

Proposition 1.9 *For* $(\theta, \gamma, \lambda) \in \mathcal{P}^0$ *and* $(i, j) \in \{1, \ldots, I-1\} \times \mathcal{I}$,

$$\partial z_i / \partial \theta_{i+1} < 0, \tag{1.43}$$

$$\partial z_{i+1} / \partial \theta_{i+1} > 0, \tag{1.44}$$

$$\partial z_j / \partial \theta_{i+1} = 0 \text{ for } j \notin \{i, i+1\}, \tag{1.45}$$

and

$$dT'(z_i; \theta_i) / d\theta_{i+1} > 0, \; dT'(z_i; \theta_{i+1}) / d\theta_{i+1} > 0, \tag{1.46}$$

$$dT'(z_{i+1}; \theta_{i+1}) / d\theta_{i+1} < 0, \; dT'(z_{i+1}; \theta_{i+2}) / d\theta_{i+1} < 0 \text{ when } i \neq I-1, \tag{1.47}$$

$$dT'(z_{i+1}; \theta_{i+1}) / d\theta_{i+1} = 0 \text{ when } i = I-1, \tag{1.48}$$

$$dT'(z_j; \theta_j) / d\theta_{i+1} = dT'(z_j; \theta_{j+1}) / d\theta_{i+1} = 0 \text{ for } j \notin \{i, i+1\}, \tag{1.49}$$

where $z \equiv g^z(\theta, \gamma, \lambda)$.

Proof. See the Appendix. ■

Increasing the productivity of the θ_{i+1}-individual does only alter his gross income and that of his nearest less productive neighbour. Indeed, by Proposition 1.6, only $\alpha_i(z_i; \theta_i, \theta_{i+1}, \gamma)$ and $\alpha_{i+1}(z_{i+1}; \theta_{i+1}, \theta_{i+2}, \gamma)$ depend on θ_{i+1}. Accordingly, the gross income levels of all other individuals, $j \neq i, i+1$, remain unaltered. As regards persons i and $i+1$, the adjustment process combines *three effects*, which are illustrated in Figure 1.5.

First, the variation in θ_{i+1} gives rise to a *local substitution effect*. The increase in person $i+1$'s productivity results in a rise in his net-of-tax wage rate, which leads him to increase his labour supply in efficiency units, z_{i+1}.

Second, changing θ_{i+1} has an *incentive effect*. As he becomes more efficient, $i+1$ has to provide less effort if he wants to imitate i. Consequently, his indifference curve through i's gross-income/consumption bundle flattens. This corresponds to an increase in the implicit marginal tax rate $T'(x_i, z_i; \theta_{i+1})$ he would face if he were cheating.

Third, i incurs an *informational externality* induced by the incentive effect. Since the cumulative social weight β_i is unaltered, the wedge $\alpha_i(z_i; \theta_i, \theta_{i+1}, \gamma)$ must stay constant (Proposition

[9] The fact that the productivity vector θ is strictly monotonically increasing ensures that (1.1) remains satisfied once a given individual productivity is changed at the margin.

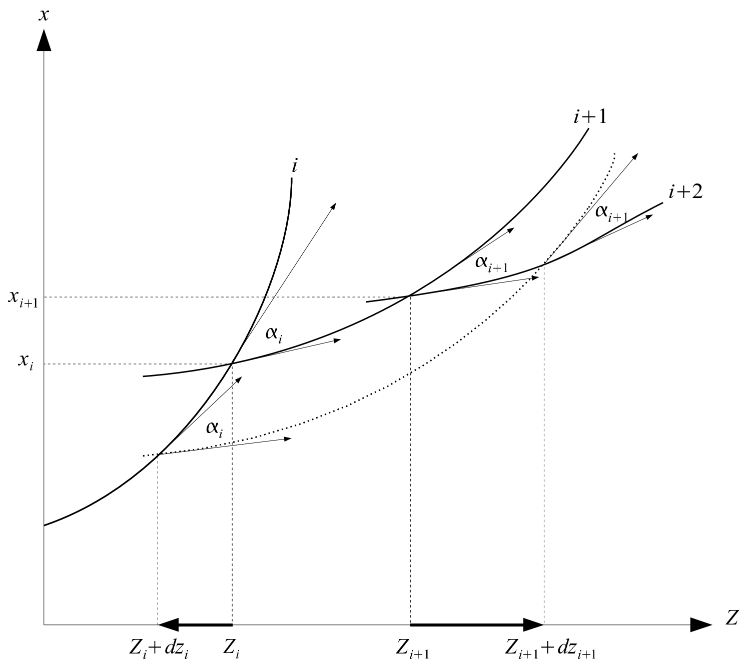

FIG. 1.5 – Effects on Gross Income of Changing θ_{i+1} at the Margin.

1.6). Consequently, the increase in $T'(x_i, z_i; \theta_{i+1})$ must be associated with an increase in the implicit marginal tax rate $T'(x_i, z_i; \theta_i)$ and thus with a reduction in i's net-of-tax wage rate. Finally, the substitution effect leads i to work less.

The changes in gross income ensure that a new monotonic chain to the left a' is obtained. However, this incentive-compatible allocation is not necessarily budget-balanced. Therefore, in a second step, all consumption levels are adjusted by a same amount in order to obtain a production-efficient allocation. This corresponds to a vertical displacement of all indifference curves through the bundles of a'. However, the direction of this displacement cannot be signed in the general case, which explains why comparative static as regards consumption is ambiguous.

1.4.3. Comparative Statics for the Social Weights

Since welfare weights in the reduced form are not a function of the skill levels, it is possible to examine how pure changes in the policy-maker tastes for redistribution alter the optimal allocation. Different kinds of changes could be considered. We concentrate herein on the scenario where the policy-maker simultaneously decides to give more weight to an individual and less to another one. In practice, social and political changes often promote greater social concern for some groups of the population while the social weight of others diminishes.

It is useful to start with the impact of changing β_i, with $i < I$.[10] It directly follows from

[10] A change in β_I is impossible since, by definition, $\beta_I \equiv 0$.

Proposition 1.6. Indeed, by (1.41), $g_j^z(\theta,\gamma,\lambda)$ is independent of β_i for $i \neq j$. Moreover, since $\alpha_i'(.;\theta_i,\theta_{i+1},\gamma)$ is strictly increasing around the optimum, (1.41) also implies that a rise in β_i reduces $g_i^z(\theta,\gamma,\lambda)$.

Proposition 1.10 *For* $(\theta,\gamma,\lambda) \in \mathcal{P}^0$ *and* $(i,j) \in \{1,\ldots,I-1\} \times \mathcal{I}$ *with* $i \neq j$, $\partial g_i^z(\theta,\gamma,\lambda)/\partial\beta_i < 0$ *and* $\partial g_j^z(\theta,\gamma,\lambda)/\partial\beta_i = 0$.

Given these preliminary results, the impact of an increase in the individual social weight of the θ_i-individual to the detriment of a more productive θ_j-individual can now be considered. Formally, the decrease in λ_j is fully compensated by an increase in λ_i, i.e. $d\lambda_i = -d\lambda_j$, while all other social weights are kept constant. By definition of $\Lambda(\theta_k)$, every β_k is increased for $k \in \{i,\ldots,j-1\}$ while all other β_k remain unaltered. By Proposition 1.10, it is thus optimal to decrease the gross income z_k of each θ_k-individual, with $k \in \{i,\ldots,j-1\}$, and to hold that of the others constant. The impact on the consumption levels and indirect utilities $V_k(\theta,\gamma,\lambda) := u(g_k^x(\theta,\gamma,\lambda), g_k^z(\theta,\gamma,\lambda);\theta_k)$ can also be signed for all $k \notin \{i,\ldots,j-1\}$.

Proposition 1.11 *Let* $(\overline{\theta},\overline{\gamma},\overline{\lambda}) \in \mathcal{P}^0$, $i \in \{1,\ldots,I-1\}$ *and* $j \in \{i+1,\ldots,I\}$. *Let* $\lambda : S \to \mathbb{R}^I$, *where* $S = (-1,1)$, *be* $\mathcal{C}^1$ *with*

$$\begin{cases} \lambda_k(0) = \overline{\lambda}_k, & k = i,j, \\ \lambda_k(s) \equiv \overline{\lambda}_k, & \forall s \in S,\ \forall k \neq i,j, \\ d\lambda_i(s)/ds = -d\lambda_j(s)/ds, & \forall s \in S. \end{cases} \tag{1.50}$$

Then, if λ_i *is increased to the detriment of* λ_j[11],

$$\begin{cases} dz_k/ds < 0, & \forall k \in \{i,\ldots,j-1\}, \\ dz_k/ds = 0, & \forall k \notin \{i,\ldots,j-1\}, \end{cases} \tag{1.51}$$

$$\begin{cases} dx_k^*/ds > 0, & \forall k < i, \\ dx_k^*/ds < 0, & \forall k \geq j, \end{cases} \tag{1.52}$$

$$\begin{cases} dV_k/ds > 0, & \forall k < i, \\ dV_k/ds < 0, & \forall k \geq j, \end{cases} \tag{1.53}$$

where $z \equiv g^z(\overline{\theta},\overline{\gamma},\overline{\lambda})$ *and* $x^* \equiv x^*(z;\overline{\theta},\overline{\gamma})$.

Proof. See the Appendix. ■

Before interpreting these results, it is worth examining the impact of this change in the social weights on the implicit optimal marginal tax rates. By (1.31), they only depend on λ through gross income z. It thus follows from (1.51) that $T'(x_k^*, z_k, \theta_k)$ and $T'(x_k^*, z_k, \theta_{k+1})$ are unaltered for $k \notin \{i,\ldots,j-1\}$ whilst $T'(x_k^*, z_k, \theta_k)$ and $T'(x_k^*, z_k, \theta_{k+1})$ are increased for $k \in \{i,\ldots,j-1\}$.

[11] The inequalities in (1.51)–(1.53) are reversed if λ_i is decreased to the benefit of λ_j.

For concreteness, let us consider that the population consists of three individuals and that person 2's social weight is increased at the expense of person 3. Let $(\overline{x}, \overline{z})$ be the initial allocation and denote by (x, z) the new one. The changes in gross income have already been explained. The adjustments in consumption can be thought of as proceeding in two steps. In the first step, the budget constraint (1.8) is left aside. By (1.51), the gross income levels of persons 1 and 3 are held fixed, i.e. $z_1 = \overline{z}_1$ and $z_3 = \overline{z}_3$, while $\overline{z}_2$ is reduced (by $d\overline{z}_2$). As a consequence, the requirement that person 3 is indifferent between his own bundle and person 2's one induces a decrease in the consumption levels $\overline{x}_2$ and $\overline{x}_3$ of both more productive individuals (by $d\overline{x}_2$ and $d\overline{x}_3$ respectively) as well as in person 3's indirect utility. Person 2's is restored to his initial indifference curve and a new monotonic chain to the left is obtained. As person 2 faces a strictly positive marginal tax rate, he reduces his consumption by a smaller amount than his gross income, i.e. $-d\overline{x}_2 > -d\overline{z}_2$. So, $d\overline{x}_3$ can be sufficiently small for

$$\sum_{i=1}^{3} \overline{x}_i - d\overline{x}_2 - d\overline{x}_3 < \sum_{i=1}^{3} \overline{z}_i - d\overline{z}_2. \tag{1.54}$$

As $\sum_{i=1}^{3} \overline{x}_i = \sum_{i=1}^{3} \overline{z}_i$, (1.54) means that the new monotonic chain to the left is not production efficient : total labour is in excess relative to total consumption. The second step consists therefore in giving

$$\varepsilon = \frac{1}{3} \left(d\overline{x}_2 + d\overline{x}_3 - d\overline{z}_2 \right) > 0 \tag{1.55}$$

additional euros of consumption to each individual. The new consumption levels are thus the following :

$$x_1 = \overline{x}_1 + \frac{1}{3} \left(d\overline{x}_2 + d\overline{x}_3 - d\overline{z}_2 \right) > \overline{x}_1, \tag{1.56}$$

$$x_2 = \overline{x}_2 - \frac{2}{3} d\overline{x}_2 + \frac{1}{3} \left(d\overline{x}_3 - d\overline{z}_2 \right), \tag{1.57}$$

$$x_3 = \overline{x}_3 - \frac{2}{3} d\overline{x}_3 + \frac{1}{3} \left(d\overline{x}_2 - d\overline{z}_2 \right) < \overline{x}_3. \tag{1.58}$$

Hence, person 1's enjoys greater consumption, contrary to person 3. The change in person 2's consumption is ambiguous. It is positive if and only if $d\overline{z}_2 < d\overline{x}_3 - 2d\overline{x}_2$. The variations in the indirect utilities directly follow from those in gross income and consumption.

1.5. CONCLUSION

Thanks to the absence of income effects on labour supply, the trade-off between equity and efficiency is very pure when individual preferences are quasilinear in consumption. This case has been investigated in depth in the continuous population version of Mirrlees model (Atkinson (1990); Piketty (1997); Diamond (1998); Salanié (1998) or d'Autume (2000)), but the analysis carried out for a finite population has concentrated on the situation where preferences are quasilinear in leisure. In this extent, the present paper contributes to filling this gap.

When preferences are quasilinear in consumption, it is not necessary to work with skilled-normalized social weights. Therefore, the respective influences of individual productivities and social weights are easier to separate in the social objective function of the reduced-form optimal income tax problem. The Spence-Mirrlees wedge is a key determinant of the properties of the optimal solution and plays an important role in signing the comparative statics. This observation is novel and is potentially useful in deriving comparative statics for any self-selection problem, not just the optimal tax problem, in which adjacent incentive constraints bind.

1.6. APPENDIX

Proof of Lemma 1.1. Differentiating $R(z_k;\theta_i,\theta_j)$ yields $R'(z_k;\theta_i,\theta_j) = v'(z_k/\theta_i)/\theta_i - v'(z_k/\theta_j)/\theta_j$ and $R''(z_k;\theta_i,\theta_j) = v''(z_k/\theta_i)/\theta_i^2 - v''(z_k/\theta_j)/\theta_j^2$. The results follow from $v'' > 0$ and $v''' > 0$ respectively. ■

Proof of Lemma 1.2. Let $(i,j,k) \in \mathcal{I}^3$ with $i<j<k$. Adding IC_{ij} and IC_{jk}, one obtains

$$x_i - v\left(\frac{z_i}{\theta_i}\right) \geq v\left(\frac{z_j}{\theta_j}\right) - v\left(\frac{z_k}{\theta_j}\right) - v\left(\frac{z_j}{\theta_i}\right) + x_k. \tag{1.59}$$

(1.59) implies that IC_{ik} holds if and only if

$$v\left(\frac{z_j}{\theta_j}\right) - v\left(\frac{z_k}{\theta_j}\right) - v\left(\frac{z_j}{\theta_i}\right) \geq -v\left(\frac{z_k}{\theta_i}\right) \Leftrightarrow R(z_j;\theta_i,\theta_j) \leq R(z_k;\theta_i,\theta_j), \tag{1.60}$$

which is satisfied since $R(.;\theta_i,\theta_j)$ is strictly increasing and $z_j \leq z_k$ by (1.16). Hence, IC_{ij} and IC_{jk} imply IC_{ik}. As a consequence, (i) $IC_{1,2}$ implies $IC_{1,k}$ for $k>2$, (ii) for $i=2,\dots,I-1$, $IC_{i,i-1}$ and $IC_{i,i+1}$ imply $IC_{i,k}$ for $k \neq i-1, i, i+1$ and (iii) $IC_{I,I-1}$ implies $IC_{I,k}$ for $k<I-1$. ■

Proof of Lemma 1.3. (i) *a is a simple monotonic chain to the left.* The proof proceeds by contradiction. Assume a is not a simple monotonic chain to the left, i.e. $u\left(x_j^*, z_j;\theta_j\right) \neq u\left(x_{j-1}^*, z_{j-1};\theta_j\right)$ for some $j \geq 2$. This is equivalent to considering that there exists $j \geq 2$ for which

$$\gamma x_j^* - v(z_j/\theta_j) > \gamma x_{j-1}^* - v(z_{j-1}/\theta_j). \tag{1.61}$$

For (1.61) to be satisfied, $(x_j, z_j) >> (x_{j-1}, z_{j-1})$. Hence, (i) is established if $z_j \leq z_{j-1}$. If $z_j > z_{j-1}$, let $\overline{x}_i = x_i^* + \varepsilon$ for $i = 1,\dots,j-1$, and $\overline{x}_i = x_i^* - \frac{j-1}{I-j+1}\varepsilon$ for $i=j,\dots,I$, where $\varepsilon>0$ is arbitrarily chosen. The new allocation $(\overline{x}, z)$ satisfies all incentive compatibility constraints for sufficiently small ε (Lemma 1.2) and is feasible because $\sum_i \overline{x}_i = \sum_i x_i^*$. In addition,

$$W(\overline{x}, z) - W(x^*, z) = \gamma \sum_{i=1}^{I} \lambda_i (\overline{x}_i - x_i^*) = \gamma \left[\sum_{i=1}^{j-1} \lambda_i \varepsilon - \sum_{i=j}^{I} \lambda_i \frac{j-1}{I-j+1}\varepsilon\right], \tag{1.62}$$

which can be minored thanks to (1.10) to get

$$W(\overline{x}, z) - W(x^*, z) \geq \gamma (j-1) \varepsilon [\lambda_{j-1} - \lambda_j] > 0, \tag{1.63}$$

contradicting $x^* \in \mathcal{X}^*(\overline{z}; \theta, \gamma, \lambda)$.
(ii) *a is production efficient.* Fix $\overline{z}$ in $\mathcal{Z}$. The constraints (1.7) are satisfied, with $z_i = \overline{z}_i$ and $x_i = x_i^*$. The proof proceeds by contradiction. Assume (1.8) is not binding. As a consequence, (1.8) is still satisfied if every x_i^* is increased by a sufficiently small $\varepsilon > 0$. This increase is incentive compatible since a same amount $\gamma\varepsilon$ is added to both sides of $IC_{i,j}$ for every $(i, j) \in \mathcal{I}^2$. A Pareto-improving change is thus feasible, contradicting $x^* \in \mathcal{X}^*(\overline{z}; \theta, \gamma, \lambda)$. ■

Proof of Proposition 1.3. It is sufficient to establish that substitution of $x^*(z; \theta, \gamma)$ into W yields $\mathcal{W}^*$ for every $z \in \mathcal{Z}$. By (1.21),

$$u(x_i^*, z_i; \theta_i) = u(x_1^*, z_1; \theta_1) + \sum_{j=1}^{i-1} R(z_j; \theta_j, \theta_{j+1}), \qquad i = 2, \ldots, I, \tag{1.64}$$

from which

$$\sum_{i=1}^{I} u(x_i^*, z_i; \theta_i) = I u(x_1^*, z_1; \theta_1) + \sum_{i=1}^{I-1} (I-i) R(z_i; \theta_i, \theta_{i+1}). \tag{1.65}$$

In addition, summing (1.4) over i on $\mathcal{I}$ and employing the equality form of (1.8),

$$\sum_{i=1}^{I} u(x_i^*, z_i; \theta_i) = \gamma \sum_{i=1}^{I} z_i - \sum_{i=1}^{I} v(z_i/\theta_i). \tag{1.66}$$

Plugging (1.66) in (1.65) and solving for $u(x_1^*, z_1; \theta_1)$,

$$u(x_1^*, z_1; \theta_1) = \frac{1}{I}\left[\gamma \sum_{i=1}^{I} z_i - \sum_{i=1}^{I} v\left(\frac{z_i}{\theta_i}\right) - \sum_{i=1}^{I-1} (I-i) R(z_i; \theta_i, \theta_{i+1})\right]. \tag{1.67}$$

Using (1.64) and (1.67),

$$\begin{aligned} W = u(x_1^*, z_1; \theta_1) \sum_{i=1}^{I} \lambda_i + \sum_{i=2}^{I} \sum_{j=1}^{i-1} \lambda_i R(z_j; \theta_j, \theta_{j+1}) &= u(x_1^*, z_1; \theta_1) I + \sum_{i=1}^{I-1} \left(\sum_{j=i+1}^{I} \lambda_j\right) R(z_i; \theta_i, \theta_{i+1}) \\ &= \sum_{i=1}^{I} \left[\gamma z_i - v\left(\frac{z_i}{\theta_i}\right) + \left(i - \sum_{j=1}^{i} \lambda_j\right) R(z_i; \theta_i, \theta_{i+1})\right], \end{aligned} \tag{1.68}$$

in which $R(z_i; \theta_i, \theta_{i+1})$ is an arbitrary number. ■

Proof of Proposition 1.4. The Lagrangian for Problem 1.2 writes

$$\mathcal{L} = \mathcal{W}^*(z; \theta, \gamma, \lambda) + \mu_1 z_1 + \mu_2 (z_2 - z_1) + \cdots + \mu_I (z_I - z_{I-1}), \tag{1.69}$$

yielding the following first-order and complementarity-slackness conditions :

$$\partial\mathcal{L}/\partial z_i = \partial\mathcal{W}^*(z;\theta,\gamma,\lambda)/\partial z_i + \mu_i - \mu_{i+1} = 0, \qquad i = 1,\ldots,I-1, \tag{1.70}$$

$$\partial\mathcal{L}/\partial z_I = \partial\mathcal{W}^*(z;\theta,\gamma,\lambda)/\partial z_I + \mu_I = 0, \tag{1.71}$$

$$\mu_1 \geq 0 \ \ (= 0 \text{ if } z_1 > 0), \tag{1.72}$$

$$\mu_i \geq 0 \ \ (= 0 \text{ if } z_i > z_{i-1}), \qquad i = 2,\ldots,I. \tag{1.73}$$

Point (i). If $i \notin \mathcal{B}^k \ \forall k$, $z_{i-1} < z_i < z_{i+1}$. Then, by (1.73), $\mu_i = \mu_{i+1} = 0$. Therefore, by (1.70), z_i satisfies $\partial\mathcal{W}^*(z;\theta,\gamma,\lambda)/\partial z_i = 0$, whose unique solution is $\widehat{z}_i$. Hence, $z_i = \widehat{z}_i$.
Point (ii). $T'(z_i;\theta_i) > 0$ for $i < I$ is established by Guesnerie and Seade (1982, Proposition 7). The remainder is shown in the text.
Points (iii)–(iv). Two kinds of bunching are to consider : (a) bunching at the bottom due to the constraint $\geqq 0$ or (b) bunching at the bottom or in the interior due to the violation of the monotonicity constraints. Assume $\{i^k,\ldots,j^k\}$ are bunched together at the optimum, with gross income $\overline{z}^k > 0$. *Case (a) :* $\mathcal{B}^k = \{1,\ldots,j^1\}$, $\mu_1 > 0$ and $\mu_{j^k+1} = 0$. Hence, summing (1.70) for $n = 1,\ldots,j^k$ yields

$$\sum_{n\in\mathcal{B}^k} \partial\mathcal{W}^*(z;\theta,\gamma,\lambda)/\partial z_n|_{z_n=\overline{z}^k} = -\mu_1 \leq 0, \tag{1.74}$$

with equality if $\overline{z}^k > 0$, which can be divided by $\#\mathcal{B}^k > 0$. *Case (b) :* $\mu_{i^k} = \mu_{j^k+1} = 0$. Summing (1.70) for $n = i^k,\ldots,j^k$ yields the equality form of (1.74). ■

Proof of Proposition 1.7. By Proposition 1.6, $g_i^z(\theta,\gamma,\lambda) = \alpha_i^{-1}(1/\beta_i;\theta_i,\theta_{i+1},\gamma)$ for $i < I$ and $g_I^z(\theta,\gamma,\lambda) = \theta_I v'^{-1}(\gamma\theta_I)$. Since $\alpha_i' > 0$ and $v'' > 0$, $g^z(\theta,\gamma,\lambda)$ is $\mathcal{C}^1$. Hence, by Property 1.2, $g^x(\theta,\gamma,\lambda)$ is also $\mathcal{C}^1$. ■

Proof of Proposition 1.8. Let $z \equiv g^z(\theta,\gamma,\lambda)$. For $i = I$, $z_I = \theta_I v'^{-1}(\gamma\theta_I)$ by (1.36) and, since $v'' > 0$, $\partial z_I/\partial\gamma > 0$. For $1,\ldots,I-1$, the first-order conditions in Proposition 1.6 write

$$\phi_i := \gamma - \frac{1+\beta_i}{\theta_i} v'\left(\frac{z_i}{\theta_i}\right) + \frac{\beta_i}{\theta_{i+1}} v'\left(\frac{z_i}{\theta_{i+1}}\right) = 0. \tag{1.75}$$

Since $v''' > 0$ and $0 < \theta_i < \theta_{i+1}$,

$$\frac{\partial\phi_i}{\partial z_i} = \frac{\beta_i}{\theta_{i+1}^2} v''\left(\frac{z_i}{\theta_{i+1}}\right) - \frac{1+\beta_i}{\theta_i^2} v''\left(\frac{z_i}{\theta_i}\right) < 0. \tag{1.76}$$

By the implicit function theorem, (1.75) defines z_i as a $\mathcal{C}^1$-function of γ, $z_i = \varphi_i^\gamma(\gamma)$, with derivative

$$\frac{\partial z_i}{\partial\gamma} \equiv \frac{d\varphi_i^\gamma(\gamma)}{d\gamma} = -\frac{\partial\phi_i/\partial\gamma}{\partial\phi_i/\partial z_i} > 0 \tag{1.77}$$

because of (1.76) and $\partial\phi_i/\partial\gamma = 1$. ■

Proof of Proposition 1.9. Let $z \equiv g^z(\theta,\gamma,\lambda)$. It is clear from Proposition 1.6 that z_j does not depend on θ_{i+1} except for $j = i, i+1$. Hence, $\partial z_j/\partial\theta_{i+1} = 0$ for $j \notin \{i, i+1\}$. Consequently,

(1.45) follows from (1.31).
It remains to examine the effect of a change in θ_{i+1} on z_i and z_{i+1}. If $\theta_{i+1} < \theta_I$, (1.75) implicitly defines z_i and z_{i+1} as $\mathcal{C}^1$-functions of θ_{i+1}, $z_i = \varphi_i^\theta(\theta_{i+1})$ and $z_{i+1} = \varphi_{i+1}^\theta(\theta_{i+1})$ respectively, with derivatives

$$\frac{d\varphi_i^\theta(\theta_{i+1})}{d\theta_{i+1}} = -\frac{\partial\phi_i/\partial\theta_{i+1}}{\partial\phi_i/\partial z_i} \text{ and } \frac{d\varphi_{i+1}^\theta(\theta_{i+1})}{d\theta_{i+1}} = -\frac{\partial\phi_{i+1}/\partial\theta_{i+1}}{\partial\phi_{i+1}/\partial z_{i+1}}. \tag{1.78}$$

(1.43) and (1.44) hold because $\partial\phi_i/\partial z_i < 0$ and $\partial\phi_{i+1}/\partial z_{i+1} < 0$ by (1.76) while

$$\frac{\partial\phi_i}{\partial\theta_{i+1}} = -\frac{\beta_i}{\theta_{i+1}^2}\left[v'\left(\frac{z_i}{\theta_{i+1}}\right) + \frac{z_i}{\theta_{i+1}}v''\left(\frac{z_i}{\theta_{i+1}}\right)\right] < 0, \tag{1.79}$$

$$\frac{\partial\phi_{i+1}}{\partial\theta_{i+1}} = \left(\frac{1+\beta_{i+1}}{\theta_{i+1}^2}\right)\left[v'\left(\frac{z_{i+1}}{\theta_{i+1}}\right) + \frac{z_{i+1}}{\theta_{i+1}}v''\left(\frac{z_{i+1}}{\theta_{i+1}}\right)\right] > 0, \tag{1.80}$$

If $\theta_{i+1} = \theta_I$, the change in z_{I-1} is obtained as previously and that in z_I comes directly from the observation that $z_I = \theta_I v'^{-1}(\gamma\theta_I)$ (Proposition 1.6) and $v'' > 0$.
Two cases must be distinguished as regards marginal tax rates. *Case (a) : $i < I-1$.* As $\partial z_i/\partial\theta_{i+1} < 0$, (1.31) and $v'' > 0$ imply an increase in $T'(z_i;\theta_i)$. Similarly, $\partial z_{i+1}/\partial\theta_{i+1} > 0$ implies a reduction in $T'(z_{i+1};\theta_{i+2})$. In addition, for $i = 1,\ldots,I-1$, (1.18) writes $T'(z_i;\theta_{i+1}) \equiv (1+1/\beta_i)T'(z_i;\theta_i)$ where $\beta_i > 0$. So, $T'(z_i;\theta_{i+1})$ increases and $T'(z_{i+1};\theta_{i+2})$ decreases. *Case (b) : $i = I-1$.* $T'(z_{i+1};\theta_{i+1})$ is unaltered, equal to zero, by Proposition 1.4. The changes in $T'(z_i;\theta_i)$ and $T'(z_i;\theta_{i+1})$ are the same as in (a). ■

Proof of Proposition 1.11. Since β_k is increased for all $k \in \{i,\ldots,j-1\}$ and unaltered otherwise, Proposition 1.10 implies (1.51). We then prove (1.52). By Proposition 1.2,

$$x_k^*(z;\theta,\gamma) = \frac{1}{I}\left[\sum_{h=1}^{I} z_h - \frac{1}{\gamma}\sum_{h=2}^{I}(I+1-h)\left(v\left(\frac{z_h}{\theta_h}\right) - v\left(\frac{z_{h-1}}{\theta_h}\right)\right)\right] + \frac{1}{\gamma}\sum_{h=2}^{k}\left[v\left(\frac{z_h}{\theta_h}\right) - v\left(\frac{z_{h-1}}{\theta_h}\right)\right], \tag{1.81}$$

the last term of which is omitted when $k = 1$.
For $k < i$, differentiating (1.81), using (1.51), and rearranging thanks to (1.73), (1.41) and (1.31),

$$\begin{aligned}\frac{dx_k^*(z;\theta,\gamma)}{ds} &= \frac{1}{I}\sum_{h=i}^{j-1}\left[1 - \frac{v'(z_h/\theta_h)}{\gamma\theta_h} + \frac{I-h}{\gamma}\left(\frac{1}{\theta_{h+1}}v'\left(\frac{z_h}{\theta_{h+1}}\right) - \frac{1}{\theta_h}v'\left(\frac{z_h}{\theta_h}\right)\right)\right]\frac{dz_h}{ds} \\ &= \frac{1}{I}\sum_{h=i}^{j-1}T'(z_h;\theta_h)\left[1-(I-h)\alpha_h(z_h;\theta_h,\theta_{h+1})\right]\frac{dz_h}{ds} \\ &= \frac{1}{I}\sum_{h=i}^{j-1}\frac{T'(z_h,\theta_h)}{\beta_h}(\beta_h - I + h)\frac{dz_h}{ds}.\end{aligned} \tag{1.82}$$

For $h = 1, \ldots, j-1$, $\beta_h - I + h < 0$ by (1.28) and (1.10), $T'(z_h, \theta_h) > 0$ by Proposition 1.4 and $dz_h/ds < 0$. Hence, $dx_k^*(z; \theta, \gamma)/ds > 0$ for $k < i$.
For $k \geq j$,

$$\frac{d}{ds}\left\{\frac{1}{\gamma}\sum_{h=2}^{k}\left[v\left(\frac{z_h}{\theta_h}\right) - v\left(\frac{z_{h-1}}{\theta_h}\right)\right]\right\} = \frac{1}{\gamma}\sum_{h=i}^{j-1}\left[\frac{1}{\theta_h}v'\left(\frac{z_h}{\theta_h}\right) - \frac{1}{\theta_{h+1}}v'\left(\frac{z_h}{\theta_{h+1}}\right)\right]\frac{dz_h}{ds} \tag{1.83}$$

$$= \sum_{h=i}^{j-1} T'(z_h, \theta_h)\,\alpha_h \frac{dz_h}{ds} = \sum_{h=i}^{j-1}\frac{T'(z_h, \theta_h)}{\beta_h}\frac{dz_h}{ds}. \tag{1.84}$$

This additional term must be added to (1.82) to get

$$\frac{dx_k^*(z; \theta, \gamma)}{ds} = \frac{1}{I}\sum_{h=i}^{j-1}\frac{T'(z_h, \theta_h)}{\beta_h}\left[\beta_h + h\right]\frac{dz_h}{ds}. \tag{1.85}$$

For $h = i, \ldots, j-1$, $\beta_h + h = \Lambda(\theta_h) > 0$, $T'(z_h, \theta_h) > 0$ by Proposition 1.4 and $dz_h/ds < 0$. Hence, $dx_k^*(z; \theta, \gamma)/ds < 0$ for $k \geq j$. (1.53) is a direct implication of (1.52) and (1.51). ■

CHAPITRE 2

WHEN KOLM MEETS MIRRLEES : ELIE

"If you have the chance to consecrate your life to thinking, you work all the time, even in your sleep." Alain Finkielkraut

2.1. INTRODUCTION[1]

Since the work by Mirrlees (1971, 1974, 1986), a large majority of economists and textbooks have adopted the second-best approach to welfarist optimal taxation. This approach discards lump-sum transfers because they are not implementable when exogenous parameters are private information. By contrast, in his last book *Macrojustice* (Kolm, 2004), Serge-Christophe Kolm proposes a tax scheme derived from fundamental principles of justice which corresponds in essence to a lump-sum tax linear in productivity. The author claims that this schedule achieves justice without implying losses in efficiency. The practicality of this proposal is likely to raise the scepticism of the common public economist for whom it is like claiming to solve the problem of squaring the circle.

However, the tax scheme derived by Kolm (2004) has many attractive features which make it worth studying further. First, it turns out to have a remarkable structure when every individual to whom it applies provides a quantity of labour in excess to what is needed for him to pay his net tax. In this case, everyone gives the fruit of the same labour duration to the society from which he receives the average donation. In other words, each citizen works a fixed fraction of his time with a view to paying his contribution to the others and is then free to devote his remaining time either to labour or leisure. This structure is referred to as *"Equal-Labour Income Equalization"* or ELIE by Kolm (2004) and appears as a very simple way of equally apportioning the common

[1] This chapter is joint work with Alain Trannoy. We have benefitted from very helpful comments received from Serge-Christophe Kolm, Antoine d'Autume, Gareth Myles and the participants in the Conference on Social Ethics and Normative Economics held in honour of Serge-Christophe Kolm at the University of Caen, May 18-19, 2007, in the Public Economics Seminar of CES–Paris 1 and ENS Cachan, and in the Economic Seminar of the University of Verona. The usual caveat applies.

contribution between the citizens, according to their means : "from each, to each other, the product of the same labour" and "from each, to each other, according to her capacities". More generally, the idea that time worked above some threshold, like the legal working hours per week, must be free of taxes has recently been defended by some economists (Godet, 2007) and is supposed to apply very soon in France.

Formally, an *ELIE tax scheme* consists of (i) a tax based on productivity but also of (ii) an *endogenous* condition on individual labour supply. Therefore, to be as clear and precise as possible, the tax in (i) should be distinguished from the ELIE tax scheme itself. The former does simply correspond to a lump-sum transfer which is linear and necessarily budget balanced when the social objective is purely redistributive. It will be referred to as *Kolm's formula* tax scheme. Given this definition, any ELIE tax scheme must belong to the family of Kolm's formula tax schemes. That is why studying Kolm's formula tax schemes in conjunction to ELIE tax schemes seems interesting to us. Nevertheless, it should be clearly emphasized that Kolm (2004) only argues in favour of ELIE, which consists of (i) and (ii).

Given these caveats, this paper aims at casting light on Kolm's formula and ELIE from the viewpoint of Mirrleesian welfarist optimal taxation so as to address some issues that are not fully embraced in Kolm's study. It endeavours to understand how the ELIE tax schemes can be interpreted in this standard framework in which they were not brought forth. This task is carried out in three steps.

First, the endogenous condition (ii) which must be added to Kolm's formula tax scheme to obtain ELIE basically depends on the labour response of the utility maximizing individuals. Therefore, it has to be checked that, when ELIE is put into practice, agents have the incentive to work not less than the time required to pay the common contribution. This means that some conditions have to be met to obtain ELIE. In addition, if one believes that a tax scheme should be universal and apply to everyone[2], restrictions on the range of possible common contributions have to be imposed. Then, the question arises to know whether these conditions are restrictive.

Second, Kolm's formula corresponds to a first best solution to the issue of wealth redistribution within the population. Given this essential feature, the two fundamental theorems of welfare economics apply. Any general equilibrium obtained from initial endowments based on Kolm's formula is Pareto optimal. Conversely, any Kolm's formula gives rise to a Pareto optimal allocation which can be obtained as the outcome of the maximization of a social welfare function, for appropriate social weights. Uncovering the weights which generate Kolm's formula and ELIE is thus of crucial relevance to the understanding of these redistributive mechanisms. The shape of these weights proves to be very specific and in sharp contrast with that considered in the standard approach to optimal taxation. In particular, it appears that these social weights must be strictly increasing with ability on the set of working people. This last property is shown to be valid for any well-behaved utility function.

Third, it is argued by Kolm (2007) that ELIE is incentive-compatible in the sense that

[2] Kolm (2004, pp. 118, 124, 127, 133-134) does not consider that ELIE should be universal. It should only apply to the "Macrojustice" case.

"it induces people to work with their full abilities". In this respect, ELIE would resolve the fundamental trade-off between equity and efficiency and justify leaving the second best for the first best. In fact, Mirrlees (1971) has popularized the idea that "the government can observe the total product of each individual, that is the product of the wage rate and the amount worked, but is unable to observe either of these alone" (Mirrlees, 1997). However, as emphasized by Mirrlees himself in the conclusion of his seminal 1971 paper, "it would be good to devise taxes complementary to the income-tax, designed to avoid the difficulties that tax is faced with". Such tax could be based on the ratio between the gross income and the hours worked by an individual. The basic difference between the latter and the Mirrleesian income tax comes from which variables are verifiable and can thus be included in the contract between the taxpayer and the policy-maker. In this respect, the incentive-compatibility of ELIE is likely to depend on whether or not hours worked are verifiable. It is shown that three kinds of tax contracts have to be distinguished. In practice, when both gross income and time duration can be observed in a second-best world, ELIE actually provides the right incentives for every agent whose labour can be timeclocked or measured by similar mechanisms. However, this is not the case for all individuals in "intellectual" occupations for which time and attendance solutions for employee labour tracking are irrelevant. It is established that these individuals have an incentive to misreport their productivity through the gross-income/labour combination they choose as soon as individual productivity is private information. Using Mirrlees's (1971) contribution, we then focus on intellectual occupations and illustrate how the first-best net transfers of ELIE can be implemented by the means of a truthful direct mechanism in weakly dominant strategies.

The article is organized as follows. Section 2 sets up the model. Section 3 investigates the maximizing behaviour of an individual facing Kolm's formula in order to illuminate some basic features of this redistributive mechanism and of ELIE. Section 4 tackles the derivation of ELIE as a first-best tax scheme in the standard framework of optimal taxation. Section 5 focuses on the implementability of ELIE. Section 6 offers concluding comments.

2.2. THE MODEL

The population consists of a continuum of individuals who only differ in productivity θ. Hence, an individual whose productivity is θ is referred to as a "θ-individual". The technology has constant returns to scale. There are two commodities, consumption and labour. Individuals have the same preferences over consumption $x \in \mathbb{R}_+$ and labour $\ell \in [0,1]$, where the time endowment of each individual has been normalized to 1. These preferences are represented by a twice continuously differentiable and concave utility function $U : \mathbb{R}_+ \times [0,1] \rightarrow \mathbb{R}$, with $U'_x > 0$, $U'_\ell < 0$ and $U(x,\ell) \rightarrow -\infty$ when $x \rightarrow 0$ or $\ell \rightarrow 1$. A θ-individual working ℓ units of time has gross income $z := \theta\ell$.

Individual productivity θ belongs to $\Theta \equiv \left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}^+$. Its cumulative density function $F : \Theta \rightarrow [0,1]$, assumed to be continuously differentiable with derivative $f(\theta) > 0$, is common knowledge. We call $\mu : 2^{\Theta} \to [0,1]$, with

$$\mu[\Omega] := \int_{\Omega} dF(\omega), \tag{2.1}$$

the measure generated by f on Θ.

The tax policy can now be introduced formally. As stressed in the introduction, we are herein interested in two kinds of tax schedules.

– Under *Kolm's formula of degree* k, every θ-individual is required to give $k\theta$ to the society from which he receives $k\mathbb{E}[\theta]$. Hence, the tax function writes $T : \Theta \times [0,1] \to \mathbb{R}$, with

$$T(\theta, k) = k(\theta - \mathbb{E}[\theta]), \tag{2.2}$$

where $\mathbb{E}[.]$ denotes the expectation over Θ.
– The *ELIE tax scheme of degree* k combines (i) Kolm's formula tax scheme of degree k with (ii) a condition on the endogenous individual labour supply. In the absence of involuntary unemployment, this condition states that all productive individuals must provide $\ell \geq k$ to take part in the overall redistributive mechanism (Kolm, 2007, p. 26-27).

A difficulty which might be raised is that, depending on individual preferences and on k, there might be a non-negligible fraction of the population choosing to provide $\ell < k$. Indeed, the *utility maximization programme* of a given θ-individual facing the ELIE tax scheme of degree k writes :

$$\max_{x \geq 0, 0 \leq \ell \leq 1} U(\theta\ell - T(\theta, k), \ell) \tag{2.3}$$

subject to the *budget constraint*

$$0 \leq x \leq \theta\ell - T(\theta, k) = \theta(\ell - k) + k\mathbb{E}[\theta]. \tag{2.4}$$

This constraint restricts the choice set of this θ-individual in two ways. First, he cannot spend more in consumption than the maximum net income he obtains when devoting his whole time endowment to labour, i.e.

$$x \leq \theta(1-k) + k\mathbb{E}[\theta] := x_{\max}(\theta, k) \tag{2.5}$$

in the absence of exogenous wealth. For $k \neq 1$, the upper bound $x_{\max}(\theta)$ is strictly increasing in θ. As a result, the more productive an individual, the wider the range of consumption levels available to him. Second, as consumption cannot be negative, a θ-individual chooses his labour supply in $[\ell_{\min}(\theta), 1]$, where

$$\ell_{\min}(\theta, k) := \max\left\{0, k\left(1 - \frac{\mathbb{E}[\theta]}{\theta}\right)\right\} < k. \tag{2.6}$$

Let $\ell^*(\theta,k)$ be the solution in ℓ to

$$\theta = -\frac{U_l\left(\theta\ell - T\left(\theta,k\right),\ell\right)}{U'_x\left(\theta\ell - k\left(\theta - \mathbb{E}\left[\theta\right]\right),\ell\right)}. \tag{2.7}$$

The *individual labour supply* $\ell(\theta,k)$ is obtained as a function of θ and k, with $\ell(\theta,k) = \ell^*(\theta,k)$ if $\ell_{\min}(\theta) < \ell^*(\theta,k) < 1$, $\ell(\theta,k) = 1$ if $\ell^*(\theta,k) \geq 1$ and $\ell(\theta,k) = \ell_{\min}(\theta,k)$ otherwise.

It can be argued that a redistributive tax schedule should be universal and a priori apply to everyone. This is actually the case in many developed countries. In France, for instance, the 13th article of the Declaration of the Rights of Man and of the Citizen, which has constitutional value, states that the common contribution should be equitably distributed among *all* the citizens in proportion to their means. That is why special attention will also be paid below to the case in which all citizens face the ELIE tax scheme of degree k. From now on, this situation is referred to as *ELIE for everyone*.

2.3. THE REQUIREMENTS OF ELIE

Since the definition of ELIE involves a condition on the endogenous labour supply, it is difficult to see which are the necessary qualifications for it to hold without using a microeconomic model. That is why, before going further, it is worth examining the utility maximization programme of the individuals facing Kolm's formula.

2.3.1. Type-Dependent Budget Sets and Corvée Labour

By definition, the *budget set* of a θ-individual is independent of any endogenous condition. It is thus the same under Kolm's formula and the corresponding ELIE tax scheme of degree k.

Given (2.4), (2.5) and (2.6), it is defined as

$$\mathcal{B}(\theta) := \left\{(x,\ell) \in [0, x_{\max}(\theta,k)] \times [\ell_{\min}(\theta,k), 1] : x \leq \theta\ell + k\left(\mathbb{E}\left[\theta\right] - \theta\right)\right\}. \tag{2.8}$$

Consequently, except in the laissez-faire case where $k = 0$ and thus $\ell_{\min}(\theta,k) = 0$ for all θ, the geometry of Kolm's formula and ELIE basically depends on whether or not an individual is less productive than the average. This is illustrated in Figure 2.1, in which the budget lines (AB) have slope θ and pass through the point $(k, \mathbb{E}[\theta])$. The fact that the individual budget set is type-dependent, with a *lower bound on labour supply* for the more productive part of the population, with $\theta > \mathbb{E}[\theta]$, is a fundamental feature of Kolm's formula and ELIE.

First, it distinguishes Kolm's formula and ELIE tax schemes from the usual tax schedules considered in the second-best optimal income tax literature. Indeed, in this literature, all individuals make their choice in the same budget set in the gross income/consumption space. In contrast, Kolm's formula and ELIE gives rise to *inequality of opportunity sets*[3] when the same

[3]Kolm has shown that ELIE nevertheless satisfies equality of opportunity for some well-defined indexes of equality of opportunity.

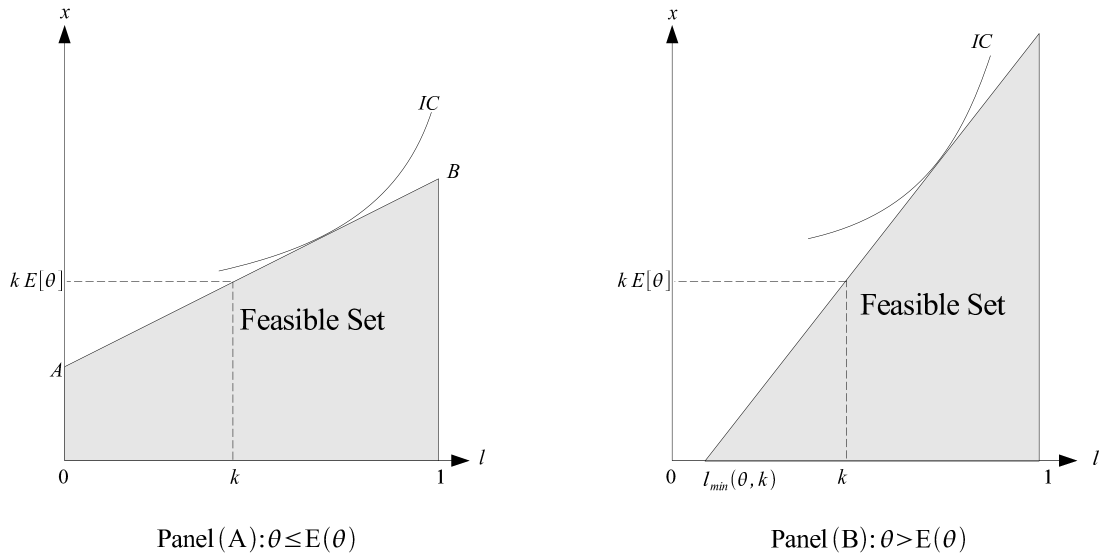

FIG. 2.1 – ELIE Case

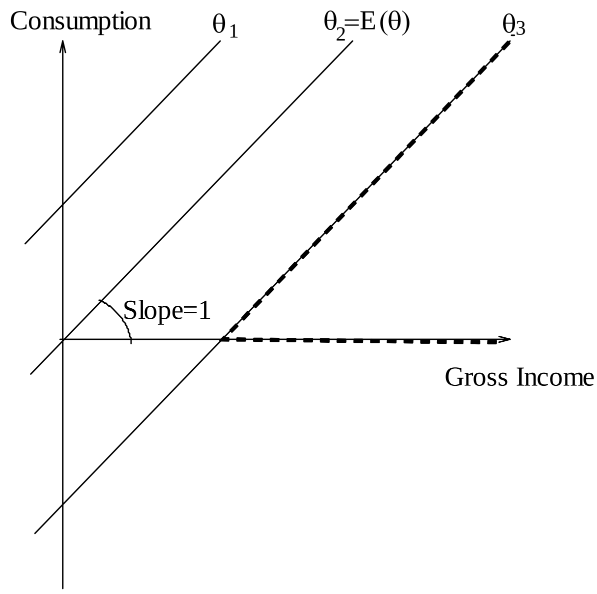

FIG. 2.2 – Unequal Opportunity Sets for Kolm's Formula. The dashed line is the boundary of $\mathcal{B}(\theta_3)$.

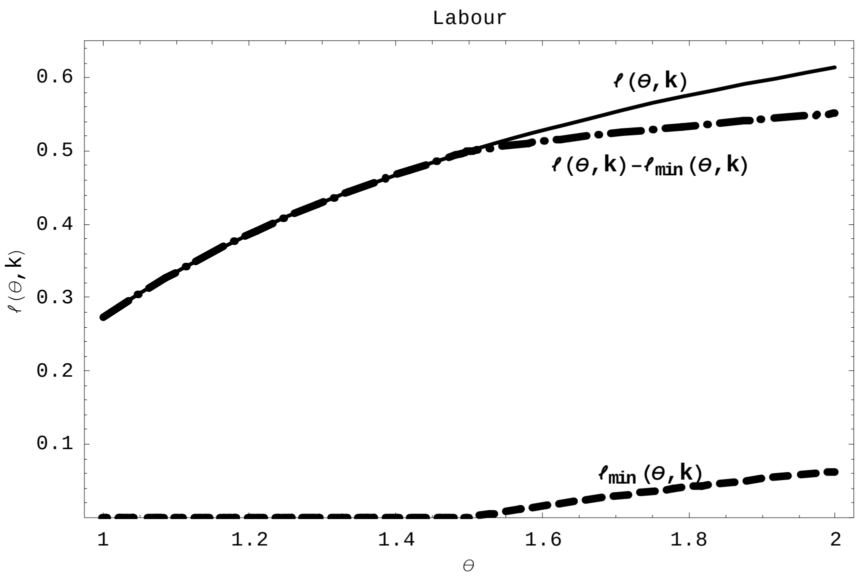

FIG. 2.3 – Corvée Labour, Free Labour, and Total Labour (Example Economy)

space is considered (cf. Figure 2.2).

Second, as regards labour supply, Kolm's formula and ELIE constrain every individual whose productivity is above the average to work at least $\ell_{\min}(\theta, k) > 0$ in order to pay the strictly positive tax $T(\theta, k) = k(\theta - \mathbb{E}[\theta])$. So, contrary to the individuals of the less productive part of the population, highly skilled individuals have not the *freedom of being idle.* This assertion must be qualified. Its validity rests on the assumption that highly skilled individuals have no assets and cannot borrow. If this assumption were relaxed, the model would no longer be static and intertemporal aspects should be taken into account.

In the absence of intertemporal dimension, $\ell_{\min}(\theta, k)$ is increased from 0 to $1 - \mathbb{E}[\theta]/\overline{\theta}$ when k goes from 0 to 1. $\ell_{\min}(\theta, k)$ can thus be regarded as unpaid labour required by Kolm's formula in lieu of taxes, i.e. as *"corvée labour"*, with k corresponding to the degree of "serfdom" of the talented.

An example is provided in Figure 2.3 in the case where individuals have preferences represented by the log-transform of the Cobb-Douglas utility function

$$U(x, \ell) = \alpha \log x + (1 - \alpha) \log (1 - \ell), \tag{2.9}$$

with $\alpha = 0.5$, productivity levels are uniformly distributed between 1 and 2, and $k = 0.25$. It appears that everyone provides an amount of labour $\ell(\theta, k)) > k$, which consists of corvée labour $\ell_{\min}(\theta, k)$ and free labour $\ell(\theta, k) - \ell_{\min}(\theta, k)$.

2.3.2. The Requirements of ELIE for Everyone

There are at least three ways of dealing with the endogeneity condition involved in the definition of the ELIE tax scheme of degree k : the class of individual preferences could be limited, each individual who would voluntarily choose to work less than the required common contribution could be excluded from the redistributive scheme, or the range of feasible equalization labour k could be restricted. The third possibility seems the most natural to us. Formally, it amounts to determining *which k are compatible with the requirement that every individual provides at least k hours of labour.*

In order to cast light on the necessary restrictions on k, it is considered that all individuals have the same preferences represented by (2.9) with $0 < \alpha < 1$. This utility specification is purely illustrative and other utility functions could naturally be considered. The θ-individual chooses the consumption/labour bundle which maximize (2.9) in $\mathcal{B}(\theta)$. The necessary and sufficient first-order condition with respect to labour yields

$$\ell(\theta) = \max\left\{0, \alpha + (1-\alpha)\frac{T(\theta,k)}{\theta}\right\}. \tag{2.10}$$

The consequences are twofold.

First, as labour supply is increasing in productivity, there exists a productivity threshold

$$\theta_0 := \frac{k}{k + \frac{\alpha}{1-\alpha}}\mathbb{E}[\theta], \tag{2.11}$$

under which individuals are idle. Therefore, the proportion of non-working people in the population is equal to $F(\theta_0)$ and all these individuals are necessarily less productive than the average, i.e.

$$\theta_0 < \mathbb{E}[\theta]. \tag{2.12}$$

Moreover, employing (2.4), the consumption of a θ-individual amounts to

$$x(\theta) = \begin{cases} \alpha(\theta - T(\theta,k)) & \text{if } \theta > \theta_0, \\ T(\theta,k) & \text{if } \theta \leq \theta_0. \end{cases} \tag{2.13}$$

Second, by (2.11), there are *no idle individuals within the population* if $\theta_0 < \underline{\theta}$, i.e. if

$$k < \min\left\{1, \frac{\alpha}{1-\alpha}\frac{\underline{\theta}}{\mathbb{E}[\theta] - \underline{\theta}}\right\} := k_0. \tag{2.14}$$

k_0 is less than 1 when $\underline{\theta} < (1-\alpha)\mathbb{E}[\theta]$. Moreover, ELIE for everyone requires

$$\begin{cases} \ell(\theta) \geq k, \ \forall \theta \in \Theta, \\ \underline{\theta} \neq 0, \end{cases} \Leftrightarrow \begin{cases} k \leq \frac{\theta}{\theta + \frac{1-\alpha}{\alpha}\mathbb{E}[\theta]}, \ \forall \theta \in \Theta, \\ \underline{\theta} \neq 0. \end{cases} \tag{2.15}$$

As the RHS of the latter inequality is increasing in θ, it follows that :

Proposition 2.1 *Under Cobb-Douglas preferences, ELIE for everyone is obtained only if*

$$k \leq \frac{\underline{\theta}}{\underline{\theta} + \frac{1-\alpha}{\alpha}\mathbb{E}[\theta]} := k_m \text{ and } \underline{\theta} \neq 0. \tag{2.16}$$

Therefore, because $k_m < k_0 \leq 1$, the individual labour response to Kolm's formula imposes restrictions on the range of equalization labour k which are compatible with ELIE for everyone. As $1-\alpha$ corresponds to the share of the Beckerian income any individual devotes to leisure, it seems reasonable to expect $\alpha \leq 0.5$. If so, k_m is less than 0.5. The condition $k \leq k_m$ might thus be regarded as restrictive. In addition, if there are individuals whose ability is zero in the population (i.e. $\underline{\theta} = 0$), an impossibility result is obtained.

2.4. KOLM'S FORMULA AND ELIE AS FIRST-BEST TAX SCHEMES

In essence, Kolm's formula (2.2) and thus the associated ELIE tax scheme of degree k can be regarded as linear taxes based on productivity, with marginal tax rate k, which are specifically designed to be budget balanced. Indeed, by construction,

$$\mathbb{E}[T(\theta, k)] = k(\mathbb{E}[\theta] - \mathbb{E}[\theta]) = 0. \tag{2.17}$$

As long as productivity is exogenous, both tax schemes pertain to the family of first-best tax schemes. The purpose of this section is to *examine what is the shape of the social weights which generate these first-best tax schemes* in a welfarist framework where individual productivity is public information.

As regards Kolm's formula, the polar situations where $k = 0$ and $k = 1$ are well-known in the literature. When $k = 0$, the laissez-faire is obtained. When $k = 1$, all individuals have the same Beckerian income $\mathbb{E}[\theta]$. This corresponds to the outcome of the maximization of a pure utilitarian social welfare function if all individuals have the same preferences represented by the utility function (2.9). In this case, there is a curse of the highly-skilled : the optimal indirect utility is strictly decreasing in productivity provided leisure is a normal good (Mirrlees, 1971, 1974). That is why it is henceforth considered that $k \in (0,1)$. In each step of our analysis, we first derive general results for Kolm's formula and then restrict the range of feasible k to apply them to ELIE.

2.4.1. Statement of the Problem

The standard viewpoint of optimal taxation is adopted. Individual social weights are given by the function $\lambda : [\underline{\theta}, \overline{\theta}] \rightarrow \mathbb{R}_{++}$, which is assumed to be $\mathcal{C}^1$. The policy-maker chooses the tax function $T : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$ which maximizes the social welfare functional

$$W = \int_{\Theta} \lambda(\theta) U(x(\theta), \ell(\theta)) dF(\theta), \tag{2.18}$$

subject to the tax revenue constraint

$$\int_{\Theta} T(\theta, k) dF(\theta) \geq 0. \tag{2.19}$$

The social objective W has been considered, among others, by Lollivier and Rochet (1983) and Weymark (1987). Since W is homogeneous of degree one in λ, $\mathbb{E}[\lambda]$ can be normalized without loss of generality. It is convenient to choose

$$\mathbb{E}[\lambda] = 1. \tag{2.20}$$

The first-best tax scheme is solution to the following optimization programme :

Problem 2.1 *Choose* $x : \Theta \to \mathbb{R}_+$ *and* $\ell : \Theta \to [0, 1]$ *to maximize* W *subject to the tax revenue constraint (2.19).*

If γ denotes the Lagrange multiplier associated with the tax revenue constraint (2.19), Problem 2.1 amounts to maximizing

$$\int_{\Theta} \{\lambda(\theta) U(x(\theta), \ell(\theta)) + \gamma[\theta\ell(\theta) - x(\theta)]\} dF(\theta). \tag{2.21}$$

The question addressed in this section can thus be summarized as follows :

Problem 2.2 *For* $k \in (0, 1)$, *find* $\lambda : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}_{++}$ *such that the optimal tax scheme* $T(\theta, k)$ *solution to Problem 2.1 corresponds to Kolm's formula.*

2.4.2. Cobb-Douglas Preferences

For clarity, it is first considered that individual preferences are represented by the cardinal logarithmic transform of the Cobb-Douglas utility function (2.9) with $0 < \alpha < 1$.

Since Problem 2.1 is concave and $f > 0$, its necessary and sufficient first-order conditions are :

$$x(\theta) = \frac{\alpha}{\gamma}\lambda(\theta), \ \forall\theta \in \Theta, \tag{2.22}$$

$$\ell(\theta) = \max\left\{0, 1 - \frac{1-\alpha}{\gamma}\frac{\lambda(\theta)}{\theta}\right\}, \ \forall\theta \in \Theta. \tag{2.23}$$

In addition, the tax revenue constraint (2.19) must be binding at the social optimum. The first-order condition (2.22) shows that consumption can be increasing in productivity if and only if the individual social weights are themselves increasing in productivity. As regards labour supply, it is known from subsection 2.3. that there exists a productivity threshold θ_0 under which everyone is idle.

Problem 2.1 corresponds to Kolm's formula only if (2.13) is also satisfied. Equalization of (2.22) and (2.13) yields :

$$\begin{cases} \lambda(\theta) = \gamma(\theta - T(\theta,k)) & \text{if } \theta > \theta_0, \\ \lambda(\theta) = \frac{\gamma}{\alpha} T(\theta,k) & \text{if } \theta \leq \theta_0. \end{cases} \quad (2.24)$$

As $\mathbb{E}[\lambda] = 1$ is constant, it follows from (2.24) and (2.14) that the unit of count in welfare γ is given by

$$\gamma = \begin{cases} 1/\mathbb{E}[\theta] & \text{if } k < k_0, \\ \frac{1}{\mathbb{E}[\theta|\theta \geq \theta_0]}\left((1-\alpha)\mathbb{E}[\lambda|\theta \geq \theta_0] + \alpha/\mu[\theta \geq \theta_0]\right) & \text{otherwise.} \end{cases} \quad (2.25)$$

where $\mathbb{E}[.|.]$ denotes the conditional expectation. Finally, using conditions (2.22) and (2.23), the tax paid paid by a θ-individual amounts to

$$T(\theta,k) = \theta\ell(\theta) - x(\theta) = \begin{cases} \theta - \frac{\lambda(\theta)}{\gamma} & \text{if } \theta > \theta_0, \\ -\frac{\alpha}{\gamma}\lambda(\theta) & \text{if } \theta \leq \theta_0. \end{cases} \quad (2.26)$$

First-Best Features of Kolm's Formula and ELIE

In order to obtain Kolm's formula of degree k, we must ensure that the tax function (2.26) is linear in θ, with slope k. Differentiating (2.26), one obtains

$$T'(\theta,k) = \begin{cases} 1 - \frac{\lambda'(\theta)}{\gamma} & if\ \theta > \theta_0, \\ -\frac{\alpha}{\gamma}\lambda'(\theta) & if\ \theta \leq \theta_0. \end{cases} \quad (2.27)$$

Therefore, the slope of the tax function is equal to k if and only if

$$T'(\theta,k) = k \Leftrightarrow \begin{cases} \lambda'(\theta) = \gamma(1-k) & if\ \theta > \theta_0, \\ \lambda'(\theta) = -\frac{\gamma}{\alpha}k & if\ \theta \leq \theta_0. \end{cases} \quad (2.28)$$

Integration of the social weights yields

$$T'(\theta,k) = k \Leftrightarrow \begin{cases} \lambda(\theta) = \gamma(1-k)\theta + \kappa_1 & if\ \theta > \theta_0, \\ \lambda(\theta) = -\frac{\gamma}{\alpha}k\theta + \kappa_0 & if\ \theta \leq \theta_0, \end{cases} \quad (2.29)$$

where κ_1 and κ_0 are constants which must be chosen so that $\mathbb{E}[\lambda(\theta)] = 1$. The social weights (2.29) can now be substituted in (2.22) and (2.23) to get

$$T(\theta,k) = \begin{cases} k\left(\theta - \dfrac{1}{\gamma}\frac{\kappa_1}{k}\right) & if\ \theta > \theta_0, \\ k\left(\theta - \dfrac{\alpha}{\gamma}\frac{\kappa_0}{k}\right) & if\ \theta \le \theta_0, \end{cases} \tag{2.30}$$

In order to obtain Kolm's formula, κ_1 and $\alpha\kappa_0$ must be equal. Given this restriction, substitution of (2.29) in (2.25) yields

$$\gamma = \frac{\kappa_1}{k\mathbb{E}[\theta]}. \tag{2.31}$$

Using (2.31), (2.29) can be rewritten as

$$T'(\theta) = k \Leftrightarrow \lambda(\theta) = \begin{cases} \kappa_1\left(\frac{1-k}{k}\frac{\theta}{\mathbb{E}[\theta]} + 1\right) & if\ \theta > \theta_0, \\ \frac{\kappa_1}{\alpha}\left(1 - \frac{\theta}{\mathbb{E}[\theta]}\right) & if\ \theta \le \theta_0. \end{cases} \tag{2.32}$$

Because $\mathbb{E}[\lambda] = 1$, it follows from (2.32) that

$$\kappa_1 = \left[\left(1 - \frac{1}{\alpha}\right)\mu[\Theta_1] + \left(\frac{1-k}{k} + \frac{1}{\alpha}\right)\frac{\int_{\Theta_1}\tau dF(\tau)}{\mathbb{E}[\theta]}\right]^{-1}. \tag{2.33}$$

When condition (2.14) holds and thus everyone works, κ_1 reduces to $\kappa_1 = k$, so that (2.32) writes

$$T'(\theta,k) = k \Leftrightarrow \lambda(\theta) = (1-k)\frac{\theta}{\mathbb{E}[\theta]} + k. \tag{2.34}$$

Kolm's Formula of Degree k : The previous results allow us to characterize the social weights under which Kolm's formula is obtained as a first-best tax scheme for $k \in (0,1)$.

Proposition 2.2 *Let $0 < \alpha < 1$ and Θ be given. Consider social weights of the form*

$$\lambda(\theta) = \begin{cases} \kappa_1\left(\frac{1-k}{k}\frac{\theta}{\mathbb{E}[\theta]} + 1\right) & if\ \theta > \theta_0, \\ \frac{\kappa_1}{\alpha}\left(1 - \frac{\theta}{\mathbb{E}[\theta]}\right) & if\ \theta \le \theta_0, \end{cases} \tag{2.35}$$

where k is in $(0,1)$ and κ_1 is defined by (2.33). Then, the first-best solution to Problem 2.1 generates a transfer scheme corresponding to Kolm's formula,

$$T(\theta,k) = k(\theta - \mathbb{E}[\theta]), \qquad \theta \in \Theta. \tag{2.36}$$

Conversely, every Kolm's formula with $k \in (0,1)$ can be generated as a first-best tax scheme.

Proposition 2.3 *Let $0 < \alpha < 1$ and Θ be given. Choose a tax scheme corresponding to Kolm's* *formula with* $k \in (0,1)$ *. Then, there exists social weights*

$$\lambda(\theta) = \begin{cases} \kappa_1 \left(\frac{1-k}{k} \frac{\theta}{\mathbb{E}[\theta]} + 1 \right) & \text{if } \theta > \theta_0, \\ \frac{\kappa_1}{\alpha} \left(1 - \frac{\theta}{\mathbb{E}[\theta]} \right) & \text{if } \theta \leq \theta_0, \end{cases} \tag{2.37}$$

with κ_1 *given by (2.33), which generate the chosen tax scheme as a first-best solution to Problem 2.1.*

The contribution of both propositions is not to establish that every Kolm's formula can be obtained as a first-best allocation since that directly follows from the first and second fundamental theorems of welfare economics, but to cast light on the *shape* of the social weights which generate a tax scheme corresponding to Kolm's formula.

Given the preference specification, it can first be noted that the social weights $\lambda(\theta)$ are (piece-wise) linear in productivity. Moreover, if there are idle individuals in the population, these social weights must be V-shaped : strictly decreasing in productivity up to θ_0, where they are continuous but non-differentiable, and then strictly increasing. Otherwise, they are strictly increasing for the whole population. This particular pattern of the social weights is in sharp contrast with that usually considered in the optimal taxation literature. Indeed, the standard view concentrates on social weights which are decreasing with ability, the rationale for this focus resting on the idea that the policy-maker is adverse to income inequality. In consequence, Propositions 2.2 and 2.3 lay the stress on a fundamental originality of Kolm's formula.

This V-shape of the social weights can be explained as follows. On the one hand, since $T(\theta, k) = k(\theta - \mathbb{E}[\theta])$, the more productive an idle individual, the less the transfer he receives. As a result, the consumption level is strictly decreasing in θ below θ_0. By (2.22), this is only possible if the social weights are strictly decreasing. On the other hand, by (2.13), consumption increases with ability above θ_0. Therefore, (2.22) implies that the social weights must be strictly increasing in productivity for the more productive part of the population.

ELIE for Everyone : By Proposition 2.1, the ELIE tax scheme of degree k is faced by everyone in the population when $k \leq k_m$ and $\underline{\theta} \neq 0$. Under these restrictions, $\theta > \theta_0$ for all θ in Θ and, by (2.33),

$$\kappa_1 = k. \tag{2.38}$$

Therefore, Propositions 2.2 and 2.3 reduce to :

Corollary 2.1 *Let* $0 < \alpha < 1$ *and* Θ *be given. Then, the first-best solution to Problem 2.1 generates a transfer scheme corresponding to ELIE if and only if*

$$\lambda(\theta) = (1-k)\frac{\theta}{\mathbb{E}[\theta]} + k. \tag{2.39}$$

2.4.3. Social Weights under ELIE for Everyone : General Case

Further insights into the shape of the social weights which generate the ELIE tax schemes are now provided when preferences are not specified. Every Pareto optimal allocation maximizes a weighted sum of utilities subject to the resource and technological constraints. The solution is assumed to be *interior* under the ELIE tax scheme of degree k, with every individual providing more than k units of labour. In this case, it is possible to employ Negishi's conditions which state that the weight of the utility of the θ-individual equals the reciprocal of his marginal utility of wealth evaluated at the supporting prices and imputed wealth (Mas-Colell, Whinston, and Green, 1995, Proposition p. 566).

Let us consider that individuals have standard well-behaved preferences for consumption and labour. By Walras' law, the consumption price can be normalized to one. Furthermore, the constant-returns-to-scale assumption implies that the wage rate of every θ-individual is fixed, equal to his productivity. Hence, in the absence of exogenous income, the wealth of every θ-individual corresponds to his Beckerian income $x_{\max}(\theta)$. His marginal utility of wealth is thus

$$U'_x\left(x_{\max}(\theta), 1\right) = U'_x\left(\theta(1-k) + k\mathbb{E}[\theta], 1\right). \tag{2.40}$$

Consequently, Negishi's conditions write

$$\lambda(\theta) = \frac{\overline{\lambda}}{U'_x\left(x_{\max}(\theta), 1\right)}, \qquad \theta \in \Theta, \tag{2.41}$$

where $\overline{\lambda}$ is a scaling factor which expresses the fact that $\lambda(\theta)$ can be normalized. As $U''_{xx} < 0$, $U'_x\left(x_{\max}(\theta), 1\right)$ is a decreasing function of θ so that the following result follows.

Proposition 2.4 *Given well-behaved preferences for consumption and leisure, the social weights which generate ELIE are strictly increasing in productivity.*

2.5. IMPLEMENTATION OF ELIE

This section focuses on the implementability of the ELIE tax schemes.

2.5.1. The Implementation Issue

Mirrlees notes in the conclusion of his seminal paper on optimum income taxation that "it would be good to devise taxes complementarity to the income tax, designed to avoid the difficulties that the tax is faced with, (...) this could be achieved by introducing a tax schedule that depends upon time worked as well as upon labour-income" (Mirrlees, 1971). In fact, once the government knows both variables in Mirrlees's framework where gross income is the product of productivity and time worked, it is capable of inferring the productivity level of each individual. So, there seems to be no reason why not designing a tax based on skill. Accordingly, Kolm (2004,

p. 175) considers that "scholars who let their thinking be directed by casual superficial remarks about difficulties of implementation and in particular information are bound to run in the wrong direction".

There are however different arguments which temper the idea that there would be a misplaced emphasis on implementation. The starting point is to distinguish occupations according to whether or not the labour time is verifiable by the employer.

On the one hand, there are jobs for which the employer can precisely record the hours that people work, thanks to a timeclock for instance. In this case, the labour time $\ell(\theta)$ can be obtained by the policy-maker, possibly at some cost. The extraction of this information belongs to the economics of tax evasion (Cowell, 1990). However, it would be incorrect to conclude from the fact that the labour supply of the θ-individual is observable that the ratio of his gross income to the number of hours worked necessarily gives his right productivity level θ. This point has been made clear by Dasgupta and Hammond (1980). Any θ-individual has the possibility to provide labour at a lower θ' productivity level. In other words, θ is the maximum productivity level of a θ-individual. Formally, let $\ell\left(\theta';\theta\right)$ be the labour supply of the θ-individual at productivity θ'. Then, his total labour supply writes $\int_{\underline{\theta}}^{\theta} \ell\left(\theta';\theta\right) d\theta'$. The computed productivity θ^c is obtained as the ratio of his total gross income $\int_{\underline{\theta}}^{\theta} \ell\left(\theta';\theta\right) \theta' d\theta'$ to his total labour supply, i.e. $\theta^c = \int_{\underline{\theta}}^{\theta} \ell\left(\theta';\theta\right) \theta' d\theta' / \int_{\underline{\theta}}^{\theta} \ell\left(\theta';\theta\right) d\theta'$. It is equal to θ if and only the tax scheme gives the θ-individual the incentive to provide all labour at his best skill $\ell(\theta;\theta)$. In other cases, taxing the computed productivity can no longer be considered as a lump-sum tax because the computed productivity is endogenous to the tax scheme! In other words, it is incorrect to conclude from the absence of cheating on time worked that the best productivity level of an individual is public knowledge. *For this type of labour, work-time evasion and implementation are both meaningful issues.*

On the other hand, there are other occupations for which it proves difficult to separate time worked from leisure. This observation notably applies to people involved in intellectual occupations for which the use of a timeclock would completely be irrelevant. As the labour duration is not verifiable, nobody can establish that a given individual is cheating. So, *the problem of work-time evasion does not exist for this second type of labour whilst the implementation issue must still be taken into account.* This last issue is particularly important because people involved in this kind of occupations are likely to belong to the more productive part of the population and pay positive taxes.

Of course, in a more general framework where productivity is the product of innate talent and effort, the knowledge of time worked would not be sufficient to identify productivity. We deliberately place ourselves in the simplest case, where effort is not taken into account, to examine if ELIE is incentive compatible.

2.5.2. Incentive Compatibility of ELIE

Incentive compatibility of ELIE basically depends on the verifiability of gross income and time worked, i.e. on *which variables can be included in the contract between the policy-maker and the agent*. By definition, a variable is verifiable if a contract that depends on it can be enforced by a third party (e.g. arbitrator, court) which can verify the value of the variable and make the parties to fulfill the contract. Hence, a contract will only depend on verifiable variables.

Keeping these remarks in mind, let us consider that individuals are faced with the ELIE tax scheme of degree k ($k \neq 0$). Three cases must be distinguished.

Non-Verifiability of Gross Income and Hours Worked

In the first case, neither gross income z nor time worked ℓ are verifiable. So, the government has no means of recovering the true productivity level of an individual from the knowledge of z and ℓ. In consequence, *the tax base is purely declaratory.* Every individual has thus an incentive to claim that his gross income z and his labour time ℓ are such that $z/\ell = \underline{\theta}$. Indeed, he then obtains the maximum transfer $-T(\underline{\theta}, k)$, which relaxes his budget constraint at the maximum. As a result,

$$\int_{\Theta} T(\underline{\theta}, k)\, dF(\theta) = k\left(\underline{\theta} - \mathbb{E}[\theta]\right) < 0, \tag{2.42}$$

which implies that the tax revenue constraint (2.4) is necessarily violated. Therefore, in this case, *the ELIE tax scheme is not incentive compatible.*

Verifiability of Gross Income and Hours Worked

The reversed case is now considered : gross income and hours worked, which are verifiable, are included in the contract between the policy-maker and the taxpayer. In this case, it is known from Dasgupta and Hammond (1980) that a tax schedule is incentive-compatible if the indirect utility is non-decreasing in productivity.

To get further insights, it is worth describing the timing of the game between the taxpayers and the policy-maker :

1. The taxpayer announces a tax schedule
$$T\left(\widehat{\theta}, k\right) = k\left(\widehat{\theta} - \mathbb{E}\left[\widehat{\theta}\right]\right). \tag{2.43}$$
2. Each θ-individual chooses at which productivity level $\widehat{\theta} = z/\ell$, with $\widehat{\theta} \leq \theta$, he wants to provide his labour. He then pays a common contribution $k\widehat{\theta}$ and receives $k\mathbb{E}\left[\widehat{\theta}\right]$ from the society.

As is standard in optimal taxation, we are interested in implementation in weakly dominant strategies. In this context, the tax schedule (2.43) is implementable if each individual cannot increase his well-being when hiding his true productivity level θ.

For convenience, it is considered that a θ-individual can only provide labour at *one* skill level $\widehat{\theta} \leq \theta$. When his gross income is z and his time worked ℓ, his observed skill is given by $\widehat{\theta} = z/\ell$. So, his utility level when he chooses $z/\ell = \widehat{\theta}$ is given by

$$\max_{\ell_{\min}\left(\widehat{\theta}\right) \leq \ell \leq 1} U\left(\widehat{\theta}\ell - T\left(\widehat{\theta}, k\right), \ell\right) \equiv V\left(\widehat{\theta}\right), \tag{2.44}$$

where $V\left(\widehat{\theta}\right)$ is the indirect utility of the $\widehat{\theta}$-individual. As a result, the θ-individual chooses to work with the skill level $\widehat{\theta}$ solution to

$$\max_{\widehat{\theta} \leq \theta} V\left(\widehat{\theta}\right). \tag{2.45}$$

Accordingly, *every individual maximizes his utility when working at his best skill as soon as* $V\left(\widehat{\theta}\right)$ *is non-decreasing in productivity.* Applying the envelope theorem to

$$V\left(\widehat{\theta}\right) \equiv U\left(\widehat{\theta}\left(\ell\left(\widehat{\theta}\right) - k\right) + k\mathbb{E}\left[\widehat{\theta}\right], \ell\left(\widehat{\theta}\right)\right), \tag{2.46}$$

one obtains

$$V'(\theta) = U'_x\left(\widehat{\theta}\left(\ell\left(\widehat{\theta}\right) - k\right) + k\mathbb{E}\left[\widehat{\theta}\right], \ell\left(\widehat{\theta}\right)\right)\left(\ell\left(\widehat{\theta}\right) - k\right). \tag{2.47}$$

Hence, since $U'_x > 0$, the indirect utility $V\left(\widehat{\theta}\right)$ is non-decreasing if and only if

$$\ell\left(\widehat{\theta}\right) \geq k, \tag{2.48}$$

which is satisfied under the ELIE tax scheme of degree k. In summary :

Proposition 2.5 *The ELIE tax scheme of degree k is implementable as a direct truthful mechanism in weakly dominant strategies when both gross income and time worked are observed by the policy-maker and verifiable.*

In this sense, "the individuals choose to work with their best skills and thus to *reveal* their capacities and to exhibit their economic value" Kolm (2007, p. 28). Hence, *ELIE is incentive compatible for all people in occupations where a "timeclock" can be used.*

Verifiability of Gross Income and Non-Verifiability of Hours Worked

It remains to examine if this result extends to the numerous individuals working in occupations for which the use of a timeclock is irrelevant. In this case, time worked is no longer verifiable so that it would be useless to include it in the contract between the policy-maker and the taxpayer. Hence, the θ-individual does no longer need to work as long as the $\widehat{\theta}$-individual if he chooses to earn the same gross income as the latter.

A natural solution in this context is to refer to some legal definition of the time worked, like the average working time for instance. Let us call it $\overline{\ell}$. The policy-maker infers that the productivity of the θ-individual whose gross income amounts to z, is $z/\overline{\ell}$. If this ratio is used as a tax base, then Kolm's tax scheme of degree k becomes

$$T\left(\frac{z}{\overline{\ell}}, k\right) = k\left(\frac{z}{\overline{\ell}} - \mathbb{E}\left[z/\overline{\ell}\right]\right) = \frac{k}{\overline{\ell}}\left(z - \mathbb{E}\left[z\right]\right). \tag{2.49}$$

The tax function (2.49) is a linear tax on gross income, with marginal tax rate $k/\overline{\ell}$ and basic income $k\mathbb{E}\left[z/\overline{\ell}\right]$. It is budget balanced since, by construction, $\mathbb{E}\left[T\left(z/\overline{\ell}\right)\right] = 0$. Moreover, it is incentive compatible provided the Spence-Mirrlees condition holds and $k/\overline{\ell} < 1$. However, it does no longer corresponds to the first-best ELIE tax scheme ; it is nothing but a linear income tax, with no specific originality.

An alternative solution is to ask every agent to report his non-verifiable time worked $\widehat{\ell}$ in conjunction with his verifiable gross income $\widehat{z}$. The inferred productivity level $\widehat{\theta} = \widehat{z}/\widehat{\ell}$ is then used as a tax base. The hidden productivity level is $\theta = \widehat{z}/\ell$, where ℓ is the actual time worked. In this case, the programme of the θ-individual is to choose

$$\left(\widehat{z}^*\left(\theta\right), \widehat{\ell}^*\left(\theta\right)\right) = \underset{\widehat{z} - T\left(\frac{\widehat{z}}{\widehat{\ell}}, k\right) \geq 0,\ 0 \leq \widehat{\ell} \leq 1}{\arg\max} U\left(\widehat{z} - T\left(\frac{\widehat{z}}{\widehat{\ell}}, k\right), \frac{\widehat{z}}{\theta}\right). \tag{2.50}$$

Hence, every θ-individual claims that he worked $\widehat{\ell}^*\left(\theta\right) = 1$ in order to maximize his utility. Since he never chooses to work 24 hours a day ($U \to -\infty$ when $\ell \to 1$), this means that he *overstates* his labour time in such a way that the policy-maker *underestimates* his productivity, i.e. $\widehat{\theta} = \widehat{z}^*\left(\theta\right) < \theta$.

Proposition 2.6 *Let gross income be verifiable and time worked be non-verifiable. Then, if the ELIE tax scheme of degree k is based on the inferred productivity level $\widehat{\theta}$, everyone has an incentive to overstate his time worked so as to understate his true productivity level θ. Therefore, this tax scheme is not implementable as a direct truthful mechanism in weakly dominant strategies.*

In this sense, despite the strictly increasing social weights which generate it in a welfarist setting, ELIE is not sufficiently favourable to the highly-skilled individuals to prevent every individual from *understating* his productivity level.

2.5.3. Implications

In practice, there are many occupations in which the labour time is not verifiable by the employer. Moreover, individuals in these occupations are in average highly skilled and, by Proposition 2.6, understate their true productivity level as soon as we depart from the first-best setting. Then, there is a difficulty of implementation of ELIE when hours worked are non-verifiable.

There are different routes to construct a second-best alternative to the ELIE tax scheme of degree k. A first route would retain the social weights (2.35) which generate Kolm's formula as a solution to Problem 2.1 and then solve the second-best problem which takes incentive-compatibility into account. A second route, which is followed in this paper and does not rest on welfarism, aims at implementing the *first-best* transfers of Kolm's formula in a second-best setting.

For illustrative purposes, it is henceforth considered that the population consists of two types of individuals who are involved in "intellectual" occupations, with respective productivities $\underline{\theta}$ and $\overline{\theta}$ ($\underline{\theta} < \overline{\theta}$).

A Second-Best Alternative to ELIE : Principles

Given the usual results of contract theory (Laffont and Martimort, 2002), one expects that when the second-best alternative to ELIE is implemented (i) the optimal tax schedule will not involve any distortion of the labour supply of the high-type individual and (ii) the only binding incentive-compatibility constraint will be that of the high-type individual. Therefore, in a two-type population, the basic idea is to examine to which extent the utility of the low type must be decreased to make the high type indifferent between his own bundle and that of the low type. For this purpose, it is sufficient to introduce a linear tax which will distort the labour supply of the low type and ensure budget balancedness through an increase in the lump-sum subsidy to the low type. For convenience, it is assumed that there are only two individuals in the population, one for each ability level. The construction of a budget-balanced and incentive-compatible allocation proceeds in four steps.

First, when the low-skilled individual faces a distortionary tax rate t on gross income and a second-best lump-sum transfer $\widetilde{T}(\underline{\theta}, k)$, his budget constraint writes

$$x(\underline{\theta}) = (1-t)z(\underline{\theta}) - \widetilde{T}(\underline{\theta}, k), \tag{2.51}$$

The first-order condition for the utility maximization programme of the low type states that his marginal rate of substitution at the optimal bundle is equal to his net-of-tax wage rate, i.e.

$$\frac{1-\alpha}{\alpha}\frac{x(\theta)}{1-\ell(\theta))} = \theta(1-t). \tag{2.52}$$

Second, the high type has no incentive to mimic the low type if the utility the former obtains at his own bundle $\left(x\left(\overline{\theta}\right), z\left(\overline{\theta}\right)\right)$ is not less than that he receives from the bundle $(x(\underline{\theta}), z(\underline{\theta}))$ of the latter, i.e. if $V\left(\overline{\theta};\overline{\theta}\right) \geq V\left(\underline{\theta};\overline{\theta}\right)$. In order to minimize the loss in efficiency, this inequality must be binding at the optimum. Therefore, the second-best allocation must satisfy

$$V\left(\overline{\theta};\overline{\theta}\right) = V\left(\underline{\theta};\overline{\theta}\right). \tag{2.53}$$

Third, the net transfer to the low-skilled individual must be the same as in the first best $T(\underline{\theta}, k)$, which ensures that the high-skilled individual pays the same tax as in the first best. Formally, this requires

$$\widetilde{T}(\underline{\theta}, k) + tz(\underline{\theta}) = T(\underline{\theta}, k), \tag{2.54}$$

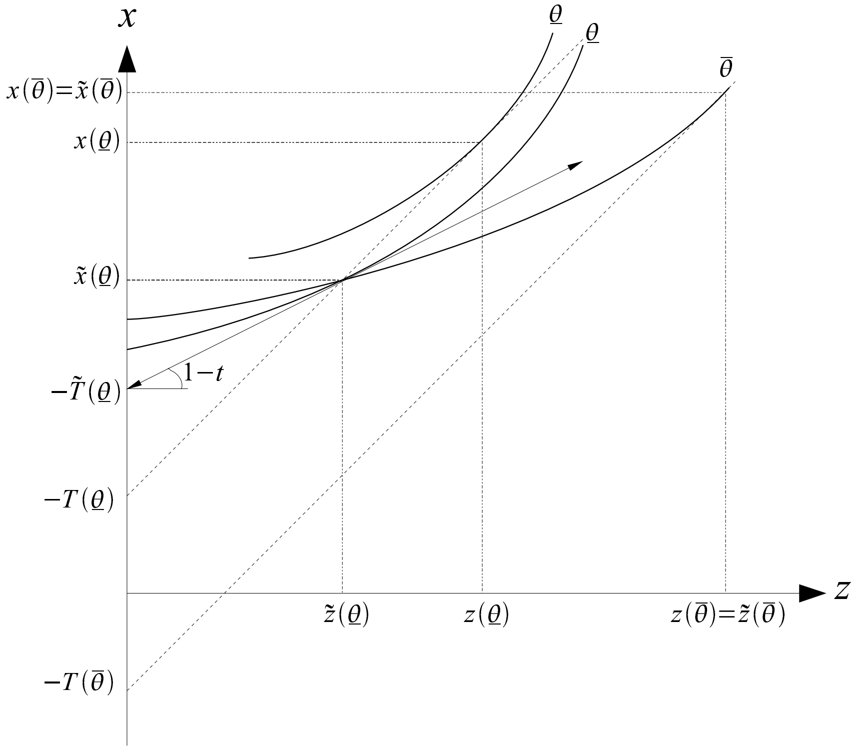

FIG. 2.4 – Construction of the Second-Best Alternative to ELIE. x, z and T correspond to first-best levels ; $\widetilde{x}$, $\widetilde{z}$ and $\widetilde{T}$ to second-best levels. Indifference curves are labelled by the productivity levels.

where $\widetilde{T}\left(\underline{\theta},k\right)$ is the second-best lump-sum tax on the low-skilled individual. In consequence,

$$\widetilde{T}\left(\underline{\theta},k\right) < T\left(\underline{\theta},k\right), \tag{2.55}$$

which means that the second-best lump-sum transfer $-\widetilde{T}\left(\underline{\theta},k\right)$ must be greater than the first best one $-T\left(\underline{\theta},k\right)$.

The fourth step consists in checking that the low type has no incentive to mimic the high type, i.e. that

$$V\left(\underline{\theta};\underline{\theta}\right) > V\left(\overline{\theta};\underline{\theta}\right). \tag{2.56}$$

The second-best problem writes therefore as follows :

Problem 2.3 *Find $\widetilde{T}\left(\underline{\theta},k\right)$ and $t \in (0,1)$ such (2.51), (2.52), (2.53), (2.54) and (2.56) are satisfied.*

Figure 2.4 shows the logic of the construction of the solution to Problem 2.3. The two budget lines for the low-skill individual cross at the new equilibrium for this individual. A simple example

FIRST-BEST

θ	x	ℓ	z	$V(\underline{\theta};\theta)$	$V(\overline{\theta};\theta)$
$\underline{\theta}$	0.531	0.469	0.469	0.531	0.397
$\overline{\theta}$	0.719	0.521	0.781	0.604	0.587

SECOND-BEST

θ	x	ℓ	z	$V(\underline{\theta};\theta)$	$V(\overline{\theta};\theta)$
$\underline{\theta}$	0.475	0.413	0.413	0.528	0.397
$\overline{\theta}$	0.719	0.521	0.781	0.587	0.587

TAB. 2.1 – First-Best and Second-Best Allocations Corresponding to ELIE

is now provided to show the practicality of this approach.

A Second-Best Alternative to ELIE : Example

A complete solution to Problem 2.3 is now provided in the case where both individuals have the same Cobb-Douglas utility function

$$U(x,\ell) = x^{\alpha}(1-\ell)^{1-\alpha}. \tag{2.57}$$

It is assumed that $k = 1/4$, $\alpha = 1/2$, $\underline{\theta} = 1$ and $\overline{\theta} = 3/2$.

In the first-best setting, the ELIE tax scheme of degree k is such that

$$T(\underline{\theta},k) = \frac{1}{4}\left(1-\frac{1}{2}\left(1+\frac{3}{2}\right)\right) = -\frac{1}{16}, \tag{2.58}$$

while $T(\overline{\theta},k) = 1/16$. As $k < k_m \simeq 0.44$, it is known from Proposition 2.1 that every individual provides labour in excess of the common contribution k. Table 2.1 gives the corresponding first-best labour supply, consumption and utility levels. Although $V(\underline{\theta};\underline{\theta}) < V(\overline{\theta};\overline{\theta})$, it is verified that the high-skilled individual has an incentive to misreport his productivity whilst the low-skilled individual reveals his type truthfully. The mimicking behaviour of the high-skilled causes a public deficit equal to $T(\underline{\theta},k) \times 2 = 1/8$ and the budget-balanced constraint is thus violated. This illustrates the result obtained in Proposition 2.6 : *the high-skilled individual does not choose to work with his best skill and thus to "reveal" his capacity and to exhibit his economic value.*

The second-best Problem 2.3 amounts to finding t and $T(\underline{\theta})$ solutions to

$$\begin{cases} V(\overline{\theta};\overline{\theta}) = x^{\alpha}(\underline{\theta})\left(1-\frac{\underline{\theta}\ell(\underline{\theta})}{\overline{\theta}}\right)^{1-\alpha}, \\ \frac{1-\alpha}{\alpha}\frac{x(\underline{\theta})}{1-\ell(\underline{\theta})} = (1-t)\,\underline{\theta}, \\ x(\underline{\theta}) = (1-t)\,\underline{\theta}\ell(\underline{\theta}) - \widetilde{T}(\underline{\theta},k), \\ \widetilde{T}(\underline{\theta},k) + t\underline{\theta}\ell(\underline{\theta}) = T(\underline{\theta},k), \end{cases} \tag{2.59}$$

where $x\left(\overline{\theta}\right) \simeq 0.719$, $\ell\left(\overline{\theta}\right) \simeq 0.521$ and $T\left(\underline{\theta}\right) = -1/16$ before checking that the low-skilled individual has no incentive to overstate his productivity level. Using the last three equations in (2.59), one obtains

$$x\left(\underline{\theta}\right) = \alpha\frac{1-t}{1-\alpha t}\left(\underline{\theta} - T\left(\underline{\theta}, k\right)\right), \tag{2.60}$$

$$\ell\left(\underline{\theta}\right) = \frac{\alpha\left(1-t\right) + \left(1-\alpha\right)T\left(\underline{\theta}, k\right)/\underline{\theta}}{1-\alpha t}, \tag{2.61}$$

which are substituted in the first equation of (2.59) to get

$$V\left(\overline{\theta};\overline{\theta}\right) = \left[\alpha\frac{1-t}{1-\alpha t}\left(\underline{\theta} - T\left(\underline{\theta}, k\right)\right)\right]^{\alpha}\left(1 - \frac{\alpha\underline{\theta}\left(1-t\right) + \left(1-\alpha\right)T\left(\underline{\theta}, k\right)}{\overline{\theta}\left(1-\alpha t\right)}\right)^{1-\alpha}. \tag{2.62}$$

It then remains to solve (2.62) in t and substitute the obtained value in (2.60) and (2.61). It is found that $t = 19.12\%$ and $\widetilde{T}\left(\underline{\theta}, k\right) = -0.1414$ instead of $T\left(\underline{\theta}, k\right) = -0.0625$. Table 2.1 provides the other second-best results. As expected, the low-skilled individual has no incentive to mimic the high-skilled one, which confirms that it was not necessary to take his incentive-compatibility constraint (2.56) explicitly into account. In addition, his second-best indirect utility is only slightly reduced compared to the first-best one ($\simeq -0.56\%$). This is a very modest price to pay for the ELIE tax scheme to induce individual truthtelling.

The clue is to deteriorate the position of the low-skilled in such a way that this decline is seen more painful by the high-skilled than by the low-skilled. It is possible to do so because they do not value a reduction in gross income in the same way. The adjustment proceeds as follows.

First, the introduction of the distortive tax gives rise to a substitution effect which induces the low-skilled to reduce his labour supply. The associated income effect impoverishes the low-skilled who is thus encouraged to work more. In general, the variation in labour supply is ambiguous whilst consumption is reduced. However, in the example, the substitution effect prevails because the income effect is partially compensated by the increase in the lump-sum transfer to the low-skilled individual. So, the low-skilled individual chooses to increase his leisure at the detriment to consumption.

Second, both individuals value the reduction in consumption in the same way because they have the same utility function. On the contrary, they do not equally value the impact of the decrease in gross income required from the low-type individual. This reduction translates into a smaller increase in leisure for the high-skill individual than for the low-skill individual since the former is more productive than the latter. Hence, the new bundle offered to the low type is viewed as less attractive by the high type up to making the latter indifferent between his own bundle and that designed for the low-skilled. On the contrary, the low-type individual regards the gain in leisure he faces as much more valuable, which helps to almost compensate the impact of his reduction in consumption. That is why it is *in fine* possible to use the distortionary tax on the gross income of the low-skilled to make ELIE incentive compatible.

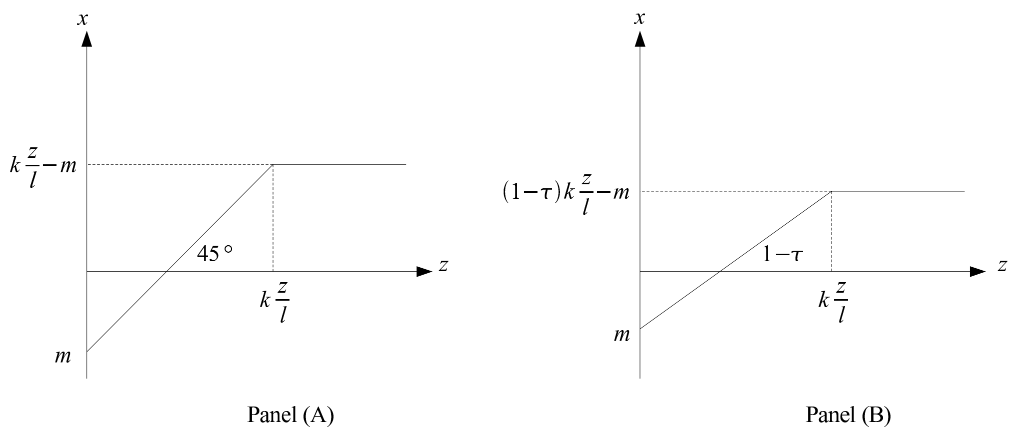

FIG. 2.5 – Panel (A) : ELIE Income Tax. Panel (B) : Overtime Exemption Flat Tax.

2.6. CONCLUDING COMMENTS

ELIE has some very attractive and striking features. It corresponds to the idea that laissez-faire should be implemented over a certain threshold of labour time, the product of this equal labour being given by everyone to all other citizens. However, it must be conceded that the ELIE tax scheme derived by Kolm are not free of weakness.

First, its definition involves an endogenous condition whose satisfaction requires qualifications. Second, it subjects the more productive part of the population to "corvée labour" and embodies therefore some troubling "feudalistic" features. Third, in the first-best framework, it is generated by social weights , which are strictly increasing in productivity for all working individuals. If Kolm's proposal should get credit for expanding the set of ethically acceptable social weights, it could nevertheless encounter stiff opposition of some normative economists because of the gross income and consumption profiles it is associated with. Fourth, it is only implementable as a truthful mechanism in weakly dominant strategies when both gross income and time worked are observable and verifiable. Since we believe that non-verifiability of time worked is a key feature of "intellectual" occupations, it seems to us that the implementation issue must be fully addressed. A simple method has been proposed so as to obtain the first-best transfers of ELIE in a second-best world where time-worked is not verifiable. This constitutes the first step of an investigation which will be carried out in a subsequent paper and would address the implementation issue for a greater number of type when there are both "timeclocked" and "intellectual" occupations.

More generally, it seems to us that the endogenous condition included in the definition of ELIE raises difficulties. For instance, it is argued by Kolm that, under ELIE, individuals benefit from a basic income $k\mathbb{E}[\theta]$ (Kolm, 2007, p. 22). Since a basic income is by definition the income received by an individual who is not working, this assertion seems problematic, at least for the more productive part of the population. For this reason, it is maybe worth adopting an alternative viewpoint from which ELIE would be closer to an income tax.

In the case *when both gross income and time worked are verifiable*, the *ELIE income tax* would write as

$$T(z, \ell; k) = \min\left\{z, k\frac{z}{\ell}\right\} - m. \tag{2.63}$$

All individuals facing the ELIE income tax would receive the same basic income m and none of them would be subject to *corvée labour*. Moreover, every hour worked above the exemption threshold kz/ℓ would be free of tax. However, this tax scheme is not incentive compatible because individuals are taxed at 100% when they work less than k. A compromise between a flat tax at rate τ and an exemption of overtime labour would be

$$T(z, \ell; \tau, k) = \tau \min\left\{z, k\frac{z}{\ell}\right\} - m, \tag{2.64}$$

which is contrasted with (2.63) in Figure 2.5. This *overtime exemption flat tax* which has the attractive features of the ELIE income tax is incentive compatible provided τ is less than 1. A more thorough study of the overtime exemption flat tax is material for further research.

CHAPITRE 3

L'IMPÔT OPTIMAL SUR LE REVENU FACE AU NOMADISME FISCAL

3.1. INTRODUCTION

Dans son article fondateur de la théorie de l'imposition optimale sur le revenu, Mirrlees (1971) considère une économie fermée. En particulier, les individus talentueux n'ont pas la possibilité d'émigrer afin d'échapper à un barème fiscal qu'ils jugeraient trop élevé. Mirrlees souligne dès l'introduction les limites de cette hypothèse dans la mesure où la menace migratoire exerce une influence significative sur la progressivité des barèmes fiscaux. Aujourd'hui, après trente-six années de mondialisation croissante, l'incidence de la mobilité des personnes sur les politiques redistributives est certainement encore plus marquée.

La plupart des travaux économiques relatifs aux migrations considèrent qu'un individu décide de résider dans le pays qui lui offre les meilleures opportunités de revenu ou de bien-être, compte tenu de ses coûts de migration éventuels (Sjaastad, 1962; Borjas, 1999). Les migrations n'en constituent pas pour autant un phénomène homogène. Elles peuvent intervenir entre un pays en développement et un pays développé ou entre deux pays également développés. Elles peuvent être motivées par le niveau de protection sociale, par le taux de salaire ou encore par des disparités de nature fiscale. Elles peuvent concerner une main d'œuvre non-qualifiée ou disposant d'un capital humain élevé. Dresser une typologie des formes migratoires s'avère par construction réducteur, les décisions individuelles de localisation résultant généralement de la combinaison de différents motifs. Il reste cependant utile, à des fins d'analyse, de dégager des formes pures de migration. Aussi, cette revue de littérature se concentrera-t-elle sur l'émigration des personnes qualifiées à des fins purement fiscales, que l'on appelera *nomadisme fiscal.*

La possibilité d'un nomadisme fiscal limite les marges de manœuvre du décideur public qui souhaite redistribuer les revenus des agents talentueux vers les moins talentueux. En effet, sous l'effet de cette contrainte, la base fiscale d'un pays donné devient endogène : les agents talentueux trop imposés qui y résident peuvent décider d'émigrer tandis que les agents vivant à l'étranger peuvent choisir d'immigrer. Les capacités de financement des programmes redistributifs sont alors affectées.

Le nomadisme fiscal n'est pas un phénomène nouveau. La Suisse ou Monaco ont, par exemple, une longue tradition d'accueil des exilés fiscaux (Le Cacheux, 2000). Néanmoins, un faisceau d'observations souligne l'importance croissance de cette forme de mobilité. Actuellement, la moitié des Français résidant à l'étranger ont des revenus supérieurs à 45 000 euros par an. Depuis l'an 2000, environ 34 000 contribuables à l'impôt sur le revenu quittent la France chaque année. Ces individus payaient avant leur départ trois fois plus d'impôts que le contribuable moyen et 70% d'entre eux ont émigré au Royaume-Uni, en Belgique, au Luxembourg, en Suisse ou vers l'Amérique du Nord, c'est-à-dire principalement vers des pays où l'imposition leur est plus favorable (DGI, 2005). En Allemagne, 145 000 contribuables qualifiés auraient quitté le pays en 2005 pour s'installer dans des pays moins redistributifs selon la Deutsche Industrie- und Handelskammertag (Chambre de commerce). Ainsi, les pays de l'Union européenne où les taux d'imposition sont élevés se trouvent confrontés à une concurrence sur leur base fiscale en raison des objectifs moins redistributifs de certains de leurs voisins (Le Cacheux and Saint-Etienne, 2005). Cette concurrence est renforcée par l'adoption d'impôts proportionnels dans de nombreux pays de l'ancienne Europe de l'Est : Lituanie, Lettonie, Estonie, Slovaquie, Roumanie, Russie, et Ukraine ont respectivement introduit des taux d'imposition uniques de 33%, 25%, 23%, 19%, 16%, 13% et 13%. La République Tchèque a décidé la création d'un impôt linéaire sur le revenu, à un taux de 15% pour 2008, abaissé à 12,5% à partir de 2009. Ce contexte renforce le poids de la variable fiscale dans les choix de localisation des agents les plus productifs.

Le nomadisme fiscal revêt une dimension spécifique. Il se distingue clairement du *brain drain* qui se fonde sur des différences de productivité entre pays (Bhagwati, 1976; Bhagwati and Hamada, 1982; Hamada, 1975). L'étude du brain drain a fait l'objet de nombreux articles, à la suite notamment de la proposition de Bhagwati consistant à autoriser les pays sources à lever un impôt sur leurs ressortissants nationaux résidant dans des pays plus développés (Bhagwati and Partington, 1976). Cette proposition reste cependant difficile à mettre en œuvre en pratique, dans la mesure où elle semble peu compatible avec le principe de réalité du droit international public, particulièrement protecteur des prérogatives régaliennnes des Etats. Le nomadisme fiscal diffère également de la fraude ou de l'évasion fiscale (Sandmo, 1981; Slemrod and Kopczuk, 2002; Chander and Wilde, 1998; Cremer and Gahvari, 1995; Cremer, Marchand, and Pestieau, 1990), qui consiste pour un agent à déclarer un revenu inférieur à celui qu'il a effectivement perçu. D'une part, les activités échappant à l'impôt contribuent tout de même à la production nationale, ce qui n'est pas le cas lorsque les individus travaillent à l'étranger. D'autre part, les gouvernements démocratiques n'ont pas la possibilité d'utiliser des instruments comparables aux contrôles et aux amendes lorsqu'ils sont confrontés au nomadisme fiscal. Face à la mobilité potentielle des agents, la politique de la carotte et du bâton (Mirrlees, 1997) cède donc le pas à la politique de l'incitation, les Etats n'ayant guère d'alternatives que d'abaisser les taux d'imposition ou laisser leur base fiscale s'expatrier.

Au cours des dernières décennies, de nombreux travaux ont examiné dans quelle mesure la mobilité des facteurs de production favorisée par le processus d'intégration économique rend plus difficile, voire impossible, la redistribution par les gouvernements nationaux (Wildasin, 1994; Epple and Romer, 1991; Cremer and Pestieau, 1998; Leite-Monteiro, 1997; Hindriks, 1999; Christiansen, Hagen, and Sandmo, 1994). Les liens entre mobilité et redistribution ont très souvent été appréhendés dans un contexte de fédéralisme fiscal. La question centrale réside alors dans l'articulation entre politiques fédérales et politiques régionales (Stigler, 1957; Cremer, Fourgeaud, Leite Monteiro, Marchand, and Pestieau, 1996; Wildasin, 1991, 1994). Ce chapitre se propose pour sa part de dresser une revue des modèles d'imposition optimale sur le revenu consacrés à l'impact de l'émigration des agents productifs, à des fins fiscales, sur le degrés de progressivité des barèmes fiscaux. Il adopte à cette fin un point de vue bien-êtriste.

De façon assez étonnante, moins d'une dizaine d'articles entrent dans notre champ d'étude (Wilson, 1980, 1982b; Mirrlees, 1982; Brewer, Saez, and Shephard, 2005; Hamilton and Pestieau, 2005; Osmundsen, 1999; Osmundsen, Schjelderup, and Hagen, 2000; Piaser, 2007; Blackorby, Brett, and Cebreiro, 2007). Tous sont confrontés à une difficulté commune. L'impact de l'émigration sur la politique de redistribution optimale ne joue pas par le seul canal de la contrainte budgétaire. Il peut également modifier l'objectif social du décideur public, ce qui rend l'analyse beaucoup plus complexe (Hamilton and Pestieau, 2005). Il s'avère alors essentiel de définir quelle est la population dont le bien-être est pris en compte par le décideur public (Mirrlees, 1982). Une première ligne de partage apparaît entre les modèles qui retiennent un objectif social invariant à la mobilité des agents et les modèles dans lesquels l'objectif social est en partie endogène. Tous les modèles, à l'exception de Hamilton and Pestieau (2005), appartiennent à la première catégorie.

Une seconde segmentation de la littérature s'opère en fonction du caractère stratégique de l'interaction entre Etats. La plupart des modèles considèrent ainsi le problème de la détermination de l'impôt optimal d'un pays, à politique fiscale du reste du monde donnée (Wilson, 1980, 1982b; Mirrlees, 1982; Brewer, Saez, and Shephard, 2005; Osmundsen, 1999). Une deuxième famille de modèles s'intéresse à la détermination des barèmes fiscaux dans plusieurs pays et étudie le processus de concurrence fiscale (Piaser, 2007) ou bien les équilibres migratoires (Hamilton and Pestieau, 2005). Une dernière approche, d'essence plus normative, pose le problème de la détermination de plusieurs barèmes fiscaux par un même décideur public, cas limite de la coopération entre Etats (Blackorby, Brett, and Cebreiro, 2007).

Les restrictions sur la forme du barème d'imposition constituent une troisième ligne de partage de la littérature. Wilson (1980, 1982b) retient une taxe linéaire, constituée d'un revenu minimum et d'un taux marginal unique ; Hamilton and Pestieau (2005); Blackorby, Brett, and Cebreiro (2007) et Piaser (2007) s'intéressent à un barème non-linéaire dans une économie à deux types, à la façon de Stiglitz (1982) ; Osmundsen (1999); Osmundsen, Schjelderup, and Hagen (2000); Mirrlees (1982) et Brewer, Saez, and Shephard (2005) considèrent une taxe non-linéaire avec une population continue.

Enfin, la distinction fondamentale entre informations publiques et privées oppose les modèles de premier rang aux modèles de second rang, qui seuls retiendront ici notre attention. Parmi ces derniers, l'asymétrie d'information porte sur le talent des agents, la préférence de localisation ou une combinaison des deux.

Notre présentation procède en cinq temps. La section 2 examine spécifiquement la question de la définition de l'objectif social en présence de mobilité. La section 3 étudie comment la mobilité des agents compétents affecte l'impôt linéaire optimal sur le revenu. La section 4 développe un modèle d'imposition non-linéaire proposé par Mirrlees dans lequel les agents ne peuvent réagir à l'impôt qu'en émigrant. La section 5 présente les modèles non-linéaires qui tiennent également compte de réponses en marges intensives. La section 6 conclut.

3.2. DE QUI PRENDRE EN COMPTE LE BIEN-ETRE ?

La mobilité des personnes pose tout d'abord le problème de la spécification de l'objectif social. En économie fermée, il est assez naturel de considérer que l'objectif social prend en compte le bien-être des ressortissants nationaux. Lorsque les individus peuvent se déplacer, les populations nationales et résidentes ne sont plus nécessairement identiques. Pour chaque Etat-Nation, on peut alors distinguer trois figures différentes : citoyen-résident, citoyen-expatrié, et résident non-citoyen. Quels sont alors les agents dont le bien-être doit être pris en compte dans la fonction de bien-être sociale d'un Etat ? Plusieurs réponses sont envisageables (Mirrlees, 1982).

L'Etat peut tenir compte du bien-être de ses citoyens, indépendamment du lieu où ils résident. Ce *critère national* trouve par exemple sa légitimité dans l'article 14 de la Déclaration des Droits de l'Homme et du Citoyen du 26 août 1789 : l'impôt, voté par le Parlement, est l'expression de la volonté générale à la détermination de laquelle chaque citoyen contribue par le droit de vote. Il est vrai cependant que l'expatriation des ressortissants nationaux s'accompagne généralement d'un relâchement des liens avec le pays d'origine. Aussi peut-on introduire un critère *citoyen-résident* qui restreint la population dont le bien-être importe aux seuls citoyens présents sur le territoire national. La logique peut également être inversée pour retenir comme pierre de touche le *critère de résidence* qui s'abstrait de l'appartenance citoyenne. Ce critère se fonde sur l'idée qu'une politique publique ne peut exclure a priori de ses objectifs ceux qui contribuent à la financer. Enfin, un dernier *critère mondial*, d'essence utopique, inclut le bien-être de tous les individus, au delà des considérations de nationalité ou de lieu de résidence.

Les critères nationaux et mondiaux présentent un avantage technique important. En effet, la population qui leur sert de support reste la même indépendamment des barèmes fiscaux mis en place. En revanche, lorsque les autres critères sont considérés, la population dont le bien-être est pris en compte est endogène à la politique fiscale dans le pays considéré et à l'étranger. Apparaît ainsi un problème de population variable. Dans ce contexte, l'utilitarisme classique ou l'utilitarisme moyen peuvent conduire à des conclusions peu attrayantes du point de vue éthique (Blackorby, Bossert, and Donaldson, 2005; Blackorby and Donaldson, 1984), justifiant l'emploi d'autres critères. Nous ne développerons pas ce point ici dans la mesure où les travaux de notre champ d'analyse se placent précisément dans des situations où ces problèmes n'apparaissent pas. Il semblait néanmoins important de les mentionner afin de souligner la nature des difficultés évitées.

3.3. L'IMPÔT LINÉAIRE

De nombreux travaux d'imposition optimale sur le revenu considèrent une taxe linéaire[1]. Un barème fiscal correspond dans cette situation à la définition d'un revenu minimum m ainsi que d'un taux marginal d'imposition constant τ. Considérer un impôt linéaire rend l'analyse techniquement plus simple puisque les contraintes d'incitation n'ont pas à être introduites explicitement dans le problème du décideur public. En outre, la définition d'un taux marginal unique est susceptible de faciliter l'administration et la collecte de l'impôt par le fisc, de rendre le droit fiscal plus transparent et d'améliorer l'assentiment des citoyens au paiement de la taxe. Wilson (1980, 1982b) a étudié l'impact de l'émigration potentielle des agents compétents sur le revenu minimum et le taux marginal d'imposition.

3.3.1. Le cadre d'analyse

Le monde se compose de deux pays, A et B. Tous les agents du pays A ont les mêmes préférences sur la consommation x et le travail ℓ, représentées par une fonction d'utilité $u(x, \ell)$ vérifiant les hypothèses usuelles de continuité et différentiabilité. Ils diffèrent par leurs niveaux de productivé θ, continûment distribué sur un intervalle $\left[\underline{\theta}, \overline{\theta}\right]$. Comme dans le problème de Mirrlees (1971), le gouvernement souhaite redistribuer les richesses. Cependant, il ne connaît que la distribution des productivités et la forme fonctionnelle des préférences. La productivité d'un agent donné constituant une information privée, l'impôt est levé sur le revenu brut, donné pour chaque individu par le produit de la productivité et du temps de travail : $z = \theta\ell$. Les agents du pays B ne jouent aucun rôle dans l'analyse. En particulier, le gouvernement de A ne tient pas compte de leur bien-être et ne pourrait les imposer s'ils vivaient en A. Dans ce cadre, le plus simple est d'imaginer que ces agents sont immobiles, par exemple en raison de coûts de migration infinis. Le gouvernement du pays A maximise une somme pondérée W de l'utilité de ses ressortissants, qu'ils vivent en A ou en B. Afin d'apprécier les effets de l'ouverture, l'argumentation se fonde sur la comparaison d'une économie ouverte et d'une économie partiellement fermée.

3.3.2. Comparaison entre économies ouverte et partiellement fermée

L'économie A est dite *ouverte* lorsque les ressortissant de ce pays peuvent résider librement en A ou en B. Pour un barème fiscal (m, τ) en A, l'individu de compétence θ maximise son utilité $u(x, \ell)$ sous la contrainte $x = m + (1 - \tau) z$, ce qui permet de définir son utilité indirecte

[1] Voir par exemple Sheshinski (1972); Atkinson (1973, 1997); Romer (1976); Dixit and Sandmo (1977); Helpman and Sadka (1978); Hellwig (1986); Tuomala (1985); Stern (1976).

$V_A(m,\tau;\theta)$, croissante en m et décroissante en τ, ainsi que l'offre de travail $z(m,\tau;\theta)$. Les niveaux d'utilité en B sont exogènes. Un individu décide de quitter A si et seulement si son utilité domestique est inférieure à son utilité à l'étranger. Cependant, à compétence donnée, tous les individus n'ont pas les mêmes opportunités s'ils choisissent d'émigrer. Une fonction $f(V_A(m,\tau;\theta);\theta)$ indique combien d'agents θ travaillent dans le pays A lorsque les paramètres fiscaux sont (m,τ). Cette fonction est supposée de classe C^1, avec $f'_V > 0$: un accroissement de l'utilité indirecte en A limite la mobilité des agents. Dans ce cadre, un accroissement de m ou une diminution de τ augmente le nombre de résidents de compétence θ puisque

$$-\frac{\partial f}{\partial V_A}\frac{\partial V_A}{\partial \tau} = \frac{\partial f}{\partial V_A}\frac{\partial V_A}{\partial m} z \geq 0. \tag{3.1}$$

L'impôt est levé sur les seuls ressortissants domiciliés en A. La contrainte budgétaire de l'Etat s'écrit donc

$$G(m,\tau) = \int_{\underline{\theta}}^{\overline{\theta}} (\tau z(m,\tau;\theta) - m) f(V_A(m,\tau;\theta);\theta)\, d\theta = 0. \tag{3.2}$$

Le problème d'imposition optimale pour l'économie ouverte A consiste donc à trouver les paramètres (m^o,τ^o) qui permettent de maximiser l'objectif social W sous la contrainte (3.2).

Il convient à présent de construire l'*économie partiellement fermée.* Considérons à cette fin l'économie ouverte précédente avec les taux optimaux d'imposition (m^o,τ^o). Supposons maintenant que les possibilités de migration disparaissent pour les agents dont la compétence appartient à un ensemble $\mathbb{P}$. L'économie obtenue est dite partiellement fermée puisqu'elle reste ouverte pour tout $\theta \notin \mathbb{P}$. Il est important de noter que les agents de $\mathbb{P}$ ne reçoivent pas de compensation pour rester en A. Puisque l'économie A est moins confrontée à la mobilité des agents, le couple (m^o,τ^o) n'est normalement plus socialement optimal. L'exercice consiste alors à étudier quelles petites modifications du barème fiscal à partir de (m^o,τ^o) permettent d'accroître le bien-être social. Le résultat formellement obtenu est assez intuitif : lorsque l'économie est partiellement fermée à des niveaux de productivité élevés, il est possible d'accroître l'utilité sociale en augmentant le taux marginal d'imposition et le revenu minimum. La conclusion principale de l'analyse apparaît donc en contrepoint : la menace de migration par des agents de haute compétence réduit les possibilités de redistribution, par l'abaissement du taux marginal qu'elle exige.

La restriction à la linéarité implique peut-être trop directement ce résultat. Le décideur public ne dispose en effet dans ce cas que d'un seul instrument de politique puisque le revenu minimum m et le taux marginal τ sont liés par la contrainte budgétaire. Ses marges d'action face à l'émigration potentielle de nombreux agents sont donc intrinsèquement limitées. Un impôt non-linéaire lui offre davantage de degrés de liberté : il peut moduler le taux marginal d'imposition en fonction du revenu pour abaisser ou élever le taux moyen d'imposition des agents qui est une variable clé du choix de localisation. Les modèles présentés par la suite relâchent tous la contrainte de linéarité de l'impôt.

3.4. LE MODÈLE DE MIRRLEES

3.4.1. Le cadre d'analyse

Mirrlees (1982) cherche à apprécier dans quelle mesure un impôt élevé encourage l'émigration. Il pose à cette fin le problème de la détermination du barème fiscal optimal dans un pays A dont les agents ont la possibilité d'émigrer vers un pays B. L'objectif social est indépendant de la politique fiscale choisie. La population consiste en un continuum d'individus qui diffèrent en productivités et propensions à émigrer. Contrairement à Mirrlees (1971), la productivité de chaque agent θ, égale au taux de salaire, est connaissance commune. L'inclination à quitter le pays est quant à elle une information privée. Le modèle ne comporte aucun arbitrage travail/loisir : chaque agent de compétence θ offre une unité de travail en l'échange de laquelle il reçoit un revenu brut $z(\theta) = \theta$. Dans ce contexte, les individus disposent d'un unique moyen d'action face au barème fiscal : l'émigration.

L'utilité dans le pays A provient de la consommation d'un bien agrégé x. Un individu de compétence θ consommant le bien en quantité $x(\theta)$ reçoit un niveau d'utilité

$$V_A(\theta) = u(x(\theta);\theta). \tag{3.3}$$

A compétence donnée, tous les individus n'ont pas la même propension à émigrer. Lorsque l'utilité est égale à $V_A(\theta)$, le nombre d'agents de compétence θ résidant en A s'élève à $f(V_A(\theta);\theta)$. On peut alors considérer que $N(\theta) - f(V_A(\theta);\theta)$ individus de productivité θ vivent en B où leur utilité est exogène, égale à $V_B(\theta)$.

Un décideur public bienveillant tient compte du bien-être de tous ses ressortissants, indépendamment de leur lieu de résidence, selon une pondération $\lambda(\theta) \in \mathbb{R}_{++}$. Seuls les individus vivant en A sont imposés. L'objectif social peut s'écrire

$$\int_\theta \lambda(\theta)\, u(x(\theta);\theta)\, f(u(x(\theta);\theta);\theta)\, d\theta + \int_\theta \lambda(\theta)\, V_B(\theta)\, [N(\theta) - f(u(x(\theta);\theta);\theta)]\, d\theta, \tag{3.4}$$

où les deux intégrales tiennent respectivement compte du bien-être des agents qui résident en A et en B, et doit être maximisé sous la contrainte budgétaire

$$\int_\theta (z(\theta) - x(\theta))\, f(u(x(\theta);\theta);\theta)\, d\theta \geq 0. \tag{3.5}$$

Il n'y a pas de contraintes d'incitation puisque l'offre de travail individuelle est fixe et observable. Le problème d'imposition optimale consiste alors à choisir un niveau de consommation $x(\theta)$ pour chaque niveau de productivité.

3.4.2. Le barème optimal

Une stratégie simple de résolution considère les effets d'une petite réforme fiscale autour d'un barème optimal. Imaginons par exemple que les agents de compétence θ voient leur niveau de consommation en A, $x(\theta)$, augmenté d'une quantité dx. En termes équivalents, leur impôt diminue de dx euros. Cette modification marginale à deux effets.

Un premier effet modifie directement le niveau de bien-être social : l'augmentation marginale de consommation accroît l'utilité des individus θ vivant en A. A partir de (3.4), on obtient :

$$dW = \lambda(\theta)\, u'_x(x(\theta);\theta)\, f(u(x(\theta);\theta);\theta) \times dx \\ + \lambda(\theta)\left[u(x(\theta);\theta) - V_B(\theta)\right] \times f'_V(V(\theta);\theta) \times dx. \quad (3.6)$$

Il semble raisonnable de considérer que les agents qui émigrent sont en fait indifférents à la marge entre A et B, ce qui fait disparaître le second terme de l'équation (3.6).

Un second effet intervient par le biais de la contrainte budgétaire en A. La hausse de consommation dx pour les agents de productivité θ doit être financée, ce qui est coûteux en termes de bien-être social. En utilisant la contrainte budgétaire (3.5), on obtient :

$$dG = \left[-f(u(x(\theta);\theta)) + (z(\theta) - x(\theta))\, f_v(u(x(\theta);\theta);\theta)\, u'_x(x(\theta);\theta)\right] \times dx. \quad (3.7)$$

Une petite réforme fiscale autour d'un barème optimal ne peut avoir d'effets de premier ordre. Aussi doit-on avoir $dW + dG = 0$, qui implique :

$$\frac{z(\theta) - x(\theta)}{x(\theta)} = \frac{1 - g(\theta)}{\eta(\theta)}, \quad (3.8)$$

avec

$$g(\theta) := \lambda(\theta) . u'_x(x(\theta);\theta), \quad (3.9)$$

$$\eta(\theta) := \frac{x(\theta)\, f_v(u(x(\theta);\theta);\theta)\, u'_x(x(\theta);\theta)}{f(u(x(\theta);\theta);\theta)}. \quad (3.10)$$

$\eta(\theta)$ correspond à l'élasticité de migration par rapport au revenu net. Dans la mesure où $z(\theta)$ est exogène, égal à θ, la formule (3.8) donne les taux d'imposition en A qui permettent de maximiser le bien-être social lorsque les agents peuvent émigrer. Cette formule dépend de deux facteurs essentiels. Le premier facteur $1/\eta(\theta)$ joue en faveur d'un abaissement du taux d'imposition pour les agents les plus mobiles. Il est donc par nature protecteur des bases fiscales potentiellement mobiles et reflète un souci d'efficacité. Le second facteur traduit des considérations éthiques. En effet, puisque $g(\theta)$ est l'utilité sociale marginale de la consommation des agents de productivité θ, il indique que le taux d'imposition doit décroître avec le poids dans l'objectif social. En général, la formule (3.8) ne permet pas une résolution explicite du problème d'imposition optimale car l'élasticité de migration dépend du revenu net $x(\theta)$ qui est endogène. En faisant l'hypothèse d'une élasticité constante, égale à 0.5, Mirrlees (1982) fournit des exemples pour lesquels des taux d'imposition optimaux restent relativement élevés. Néanmoins, ces résultats restent avant tout illustratifs, notamment parce que la réaction des agents à l'impôt est limitée au choix de localisation.

3.4.3. Taux d'imposition maximal des plus talentueux

Dans le modèle précédemment exposé, les individus opèrent un choix binaire entre rester dans le pays où ils résident ou émigrer, ce qui correspond conceptuellement à un choix en marges extensives (voir Saez (2002); Choné and Laroque (2005); Laroque (2005) par exemple). Brewer, Saez, and Shephard (2005, p. 25) retrouvent ainsi très exactement la formule (3.8) en partant d'un modèle d'économie fermée où les agents ont la possibilité d'entrer ou non sur le marché du travail (Saez, 2002). Il suffit pour ce faire de remplacer l'élasticité de la population active de compétence donnée au taux d'imposition par l'élasticité de migration. Cette façon de procéder ne permet pas cependant d'apprécier complètement la spécificité de l'impact migratoire.

La contribution de Brewer, Saez, and Shephard (2005, p. 25) réside dans l'enrichissement de la formule (3.8) pour les agents les plus compétents de la population. Elle s'appuie sur la possibilité d'approximer les taux marginaux des individus les plus riches dans le modèle non-linéaire par un barème linéaire (Saez, 2001). Elle permet d'introduire dans l'analyse les réponses en marges intensives aux côtés des réponses en marges extensives. L'analyse est simplifée par l'exclusion des effets de revenu sur l'offre de travail, comme chez Diamond (1998). Supposons que le décideur public mette en place un impôt linéaire sur le revenu au-delà d'un revenu $\overline{z}$ et appelons z le revenu moyen déclaré par les contribuables concernés. Il est alors possible d'utiliser une petite réforme fiscale autour du barème optimal pour calculer le taux moyen qui maximise le bien-être social. Notons e l'élasticité hicksienne moyenne des agents de productivité supérieure à $\theta_{\overline{z}}$, $g(\theta_{\overline{z}})$ le poids social moyen des individus disposant de revenus supérieurs à $\overline{z}$ et désignons par

$$a := \frac{z}{z - \overline{z}} \tag{3.11}$$

une mesure de l'épaisseur de la distribution des revenus au-dessus de $\overline{z}$. Les réponses en marges intensives aboutissent à un taux optimal

$$\tau^* = \frac{1 - g(\theta_{\overline{z}})}{1 - g(\theta_{\overline{z}}) + a.e}. \tag{3.12}$$

Lorsque les agents peuvent émigrer, la formule (3.12) est corrigée par l'ajout de l'élasticité migratoire moyenne des agents du haut de la distribution, $\eta(\theta_{\overline{z}})$, ce qui donne

$$\tau^{**} = \frac{1 - g(\theta_{\overline{z}})}{1 - g(\theta_{\overline{z}}) + a.e + \eta(\theta_{\overline{z}})}. \tag{3.13}$$

En fixant $g(\theta_{\overline{z}}) = 0$, on obtient alors simplement la borne supérieure d'imposition pour les

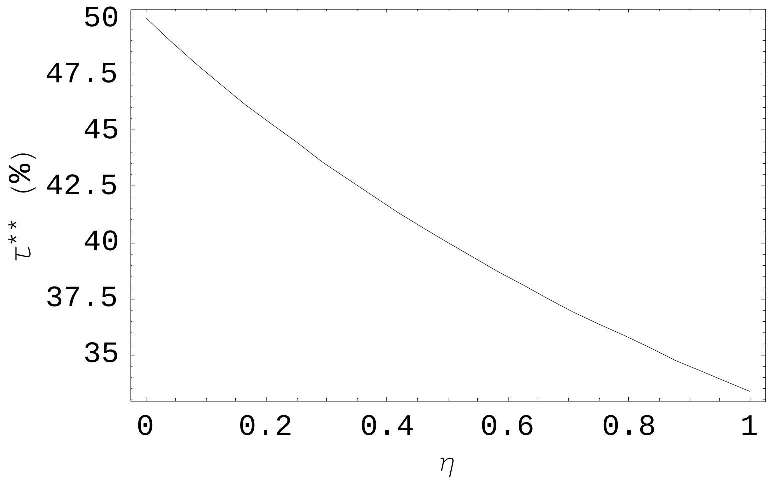

FIG. 3.1 – Taux marginal d'imposition maximal des individus les plus compétents en fonction de l'élasticité migratoire η ($e = 0.5$ et $a = 2$)

individus les plus riches. Certaines données suggèrent que l'élasticité migratoire du Royaume-Uni serait de l'odre de $\eta = 0.22$ pour la période allant de 1970 à 2000. Les estimations de Saez (2002) donnent une valeur proche de $a = 2$ en ce qui concerne les Etats-Unis, et on peut retenir ce même chiffre à titre indicatif. Enfin, une élasticité de l'offre de travail égale à $e = 0.5$ ne paraît pas déraisonnable en ce qui concerne le Royaume-Uni. On obtient dans ce cas un taux maximal d'imposition pour les individus les plus riches de $\tau^{**} = 45\%$. Pour une élasticité migratoire beaucoup plus forte de $\eta = 0.44$, le taux maximal en économie ouverte est de $\tau^{**} = 40\%$ (cf. Figure 3.1). En économie fermée, ce taux serait de $\tau^{*} = 50\%$.

La formule (3.13) fournit des indications précieuses pour les plus hauts niveaux de revenu. Son extension à une partie plus large de la population soulève différentes difficultés. Il n'est tout d'abord plus possible de supposer que le barème non-linéaire peut être approximé par un barème linéaire, ce qui rend nécessaire la prise en compte explicite des contraintes d'incitation. Les effets de revenu sur l'offre de travail sont également susceptibles de jouer davantage. Enfin, la dimension éthique apparaîtra sous une forme plus complexe.

3.5. L'IMPÔT NON-LINÉAIRE AVEC ARBITRAGE TRAVAIL/LOISIR

Jusqu'ici, aucun des modèles abordés n'introduit de contraintes d'incitation, en raison de la linéarité de l'impôt ou de l'adoption d'un raisonnement en marges extensives. Ces contraintes doivent être prises en compte explicitement lorsque l'on étudie un impôt non-linéaire en présence d'arbitrages travail/loisir. Trois axes d'analyse retiendront notre attention. Le premier d'entre eux tient compte de l'impact de la mobilité des agents sur l'objectif social (Hamilton and Pestieau, 2005). Il envisage donc des situations dans lesquelles les effets de l'émigration ne se cantonnent pas aux contraintes budgétaires ou incitatives. Une seconde ligne de recherche tente d'appréhender les conséquence d'un jeu concurrentiel portant sur des barèmes non-linéaire (Piaser, 2007). Enfin, un dernier axe se détâche des précédents en envisageant le problème de la coopération fiscale en présence de mobilité (Blackorby, Brett, and Cebreiro, 2007).

3.5.1. Prise en compte de l'impact de la mobilité sur l'objectif social

La mobilité des agents modifie la distribution des compétences dans un pays donné, ce qui est susceptible de changer la progressivité du barème d'imposition optimale (Diamond, 1998) à travers deux canaux : la contrainte budgétaire, d'une part, et l'objectif social lui-même d'autre part. Le second effet est a priori particulièrement complexe. Aussi, Hamilton and Pestieau (2005) procèdent-ils en deux temps afin de mieux le cerner.

Dans une première étape, ils étudient l'allocation optimale dans une économie à deux types d'agents où l'objectif social correspond soit au maximin soit au maximax. Ils dérivent l'utilité indirecte pour chacun des types en fonction de la proportion d'agents de haute compétence n. Cette analyse est utilisée dans un second temps afin d'examiner les effets de la mobilité d'*un* type donné d'agents sur une petite économie A appartenant à un vaste marché commun du travail B, à l'exemple d'un petit pays au sein de l'Union européenne. Les individus de la petite économie comparent leur utilité domestique et leur utilité sur le marché commun, en supposant leur décision sans effet sur les niveaux d'utilité. Les individus du marché commun procèdent à l'arbitrage inverse. Les agents décident alors d'émigrer ou d'immigrer. L'objectif social en A est le maximin lorsque les agents qualifiés sont minoritaires ($n < 1/2$) et le maximax lorsque ces derniers sont majoritaires ($n > 1/2$). L'impact de la mobilité sur l'objectif social est donc pris en compte à travers une règle de vote majoritaire. Ce raisonnement permet d'appréhender le double impact de la mobilité des agents.

Revenons maintenant plus en détail sur le cadre d'analyse considéré. Le modèle s'appuie sur la version à deux types du modèle de Mirrlees (1971) développée par Stiglitz (1982). Les agents ont en outre des préférences séparables en consommation x et travail ℓ, linéaire en travail, comme chez Lollivier and Rochet (1983) ou Weymark (1986b, 1987), représentées par la fonction d'utilité

$$U(x, \ell) := \ln x - \ell. \tag{3.14}$$

Tout les effets de revenu portent donc sur l'offre de travail, ce qui permet d'obtenir des formes réduites simples. Notons θ_2 la productivité des bas types et θ_1 celle des hauts types ($\theta_1 > \theta_2$). Le revenu brut z d'un agent est égal au produit de son temps de travail et de sa productivité, $z := \theta\ell$. Le décideur public maximise l'utilité des bas types (maximin) ou des hauts types (maximax) sous la contrainte budgétaire de l'Etat

$$n(z_1 - x_1) + (1 - n)(z_2 - x_2) \geq 0 \tag{3.15}$$

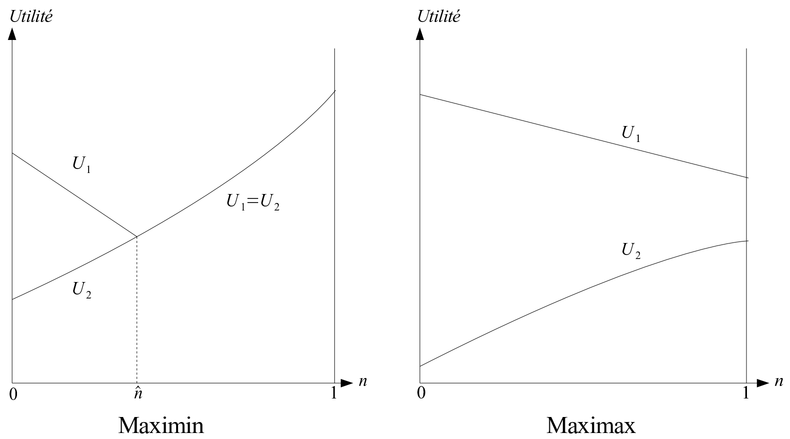

FIG. 3.2 – Profil d'utilité par type en fonction de la proportion d'agents compétents n

et les conditions usuelles d'incitation

$$\ln x_1 - \frac{z_1}{\theta_1} \geq \ln x_2 - \frac{z_2}{\theta_1}, \tag{3.16}$$

$$\ln x_2 - \frac{z_2}{\theta_2} \geq \ln x_1 - \frac{z_1}{\theta_2}. \tag{3.17}$$

On obtient alors les profils d'utilité représentés sur la Figure 3.2 où U_i désigne l'utilité indirecte, à l'optimum social, d'un agent de type θ_i, $i = \{1, 2\}$.[2]

La statique comparative des utilités indirectes à l'optimum permet d'aborder le second temps du raisonnement présenté plus haut. Nous nous concentrons ici sur les effets de la mobilité des hauts types. Appelons n_0 la proportion initiale d'agents talentueux dans la petite économie A et désignons par $\overline{U}_1$ l'utilité qu'ils peuvent obtenir en B. Afin de simplifier l'exposé, on supposera $\widehat{n}$ supérieur à $1/2$. Dans ce cas, l'utilité en A des individus compétents varie en fonction de n ; elle est décroissante pour $n < 1/2$ et $n > 1/2$, la partie de droite du graphe étant au dessus de celle de gauche.

On peut alors distinguer quatre régimes stables d'équilibre migratoire en fonction des valeurs respectives de n_0 et de n_β, défini comme le niveau de productivité pour lequel $U_1 = \overline{U}_1$. Commençons par les situations dans lesquelles l'objectif social en A est initialement le *maximin* ($n_0 < 1/2$). Lorsque $n_\beta < 1/2$, la proportion d'agents qualifiés en A converge vers n_β et U_1 vers $\overline{U}_1$. A l'inverse, si $n_\beta > 1/2$, tous les agents qualifiés vont quitter A pour recevoir leur utilité $\overline{U}_1$ en B. La population de A se composera alors en définitive uniquement de bas types. Considérons à présent l'objectif social *maximax* ($n_0 > 1/2$). Pour $n_\beta < 1/2$, tous les agents du reste du monde immigrent en A où leur utilité est supérieure à $\overline{U}_1$. L'utilité des hauts types en A tend par conséquent vers $U_1(1)$, qui s'obtient lorsque la population se compose uniquement d'agents

[2] La non-monotonie de U^1 dans le cas du maximin résulte de la solution en coin ($\ell = 0$) qu'adoptent les moins talentueux lorsque la proportion d'agents talentueux dépasse $\widehat{n}$.

de haut type. Enfin, si $n_\beta > 1/2$, n_β constitue un équilibre stable et l'utilité des agents qualifiés en A s'égalise à leur utilité de réservation.

Ces résultats sont ensuite approfondis en considérant que le reste du monde B se compose d'un grand nombre de petits pays. Il apparaît alors que la mobilité parfaite des qualifiés n'élimine pas inéluctablement les possibilités de redistribution vers les agents peu talentueux. Supposons que les agents les moins talentueux, de compétence θ_2, soient les plus nombreux dans la population mondiale et qu'ils offrent une quantité de travail positive. Il existe alors deux types d'équilibre. Dans le premier cas, la répartition de la population entre bas et haut types est la même dans tous les pays et donc l'objectif social est partout le maximin. Dans le second cas, la population de certains pays se compose uniquement de bas types tandis que d'autres pays comportent à la fois des bas types et des hauts types, avec un objectif social maximax favorable à ces derniers. La réalisation d'un équilibre symétrique ou asymétrique dépend de la répartition initiale des agents entre pays. Si les agents peu compétents représentent la majorité dans tous les pays, les agents compétents vont émigrer vers les pays où la composition de la population est le moins en leur défaveur (i.e. vers les pays où $1-n$ est relativement faible). Ce processus conduit à une structure de population homogène dans les différents pays, qui tous adoptent par conséquent le maximin. Supposons, à l'inverse, que les haut types forment la majorité des électeurs dans certains pays (au moins un). Les minorités compétentes du reste du monde vont avoir une incitation à émigrer vers ces pays. On obtient ainsi l'équilibre asymétrique.

Ce modèle permet donc de dégager dans quels cas l'objectif social reste stable, donné par le maximin ou le maximax, en dépit des mouvements migratoires qui affectent le processus de vote majoritaire dont il est issu. L'analyse s'appuie bien sûr ici sur le fait qu'il n'existe que deux objectifs sociaux, revêtant chacun une forme très simple, et sur la quasi-linéarité en loisir des préférences qui permet d'obtenir des solutions explicites des barèmes de taxation optimale. Du point de vue théorique, elle met l'accent sur l'impact de la mobilité des agents sur la forme de l'objectif social, dimension jusqu'alors passée sous silence dans les modèles d'imposition optimale.

3.5.2. Concurrence versus coopération fiscales

Un certain nombre de travaux s'intéressent à la compétition fiscale sur des barèmes optimaux non-linéaires. Piaser (2007) considère ainsi une version à deux pays A et B du modèle de Stiglitz (1982) dans laquelle les individus ont des préférences quasi-linéaires en consommation. Les agents de chacun des pays ne peuvent émigrer qu'une fois, le modèle étant statique. Ils prennent cette décision si l'utilité qu'ils reçoivent à l'étranger est supérieure à leur utilité domestique, compte tenu de leurs coûts de migration. Ces coûts de migration dépendent essentiellement du type. Les pays A et B ont les mêmes populations initialement. Les résultats les plus significatifs sont obtenus lorsque les deux pays adoptent le maximin comme objectif social et que seuls les agents compétents sont mobiles au niveau international. L'équilibre de Nash est retenu comme critère d'équilibre. Dans cette situation, les gouvernements de A et B souhaitent retenir leurs agents qualifiés et attirer ceux vivant à l'étranger afin d'augmenter leurs recettes fiscales et redistribuer davantage aux moins talentueux. Mais pour ce faire, ils doivent diminuer le taux d'imposition des talentueux. Les autres résultats obtenus sont moins transparents. Ces conclusions en demi-teinte illustrent principalement la difficulté à poser le problème de la détermination simultanée de barèmes d'imposition optimaux non-linéaires par des gouvernements en concurrence sur leurs bases fiscales.

L'adoption d'un point de vue coopératif rend le modèle plus maniable. Blackorby, Brett, and Cebreiro (2007) étudient une économie formée de deux régions A et B administrées par un même décideur public bienveillant. Ces régions sont habitées par des habitants de deux niveaux de productivité, $\theta_1 > \theta_2$. Les individus se distinguent en outre par un paramètre de préférence résidentielle $\beta \in [0,1]$ qui exprime leur attachement à la région A. Par conséquent, chaque agent est caractérisé par une paire (θ, β) avec $\theta \in \{\theta_1, \theta_2\}$ et $\beta \in [0,1]$. La préférence résidentielle est distribuée au sein de chacun des types selon des fonctions de répartition $G_1(\beta)$ pour le type θ_1 et $G_2(\beta)$ pour le type θ_2. En plus du lieu de résidence, les préférences individuelles dépendent également de la consommation x et du temps de travail ℓ. Si on appelle $\mathcal{A}$ et $\mathcal{B}$ l'ensemble des travailleurs résidant dans les régions A et B respectivement, ces préférences sont représentées par une fonction d'utilité séparable

$$U(x, \ell, \beta) = \begin{cases} v(x, \ell) + h(\beta), & \theta \in \mathcal{A} \\ v(x, \ell), & \theta \in \mathcal{B}, \end{cases} \tag{3.18}$$

où $h(\beta)$ est une fonction croissante de β. Le revenu brut d'un agent θ est donné par $z = \theta\ell$.

L'écriture spécifique des contraintes d'incitation retiendra principalement notre attention. Le décideur public à la tête des deux régions ne peut observer le couple (θ, β) pour un individu particulier. Dans ce contexte, il propose un barème d'imposition sur le revenu pour chacune des régions afin de maximiser une fonction de bien-être sociale dépendant de l'utilité de tous les agents indépendamment de leur lieu de résidence. Le décideur public ne cherche pas spécifiquement à retenir une catégorie d'agents dans une région donnée. Il n'introduit donc pas de contrainte de participation garantissant à un agent θ_i une utilité plus forte en A qu'en B ou inversement. En revanche, il doit s'assurer afin de mettre en place des barèmes révélateurs que (i) les agents d'un type θ_i donné vivant dans la région j n'ont pas intérêt à imiter les agents θ_{-i} de la région j mais également que (ii) les agents θ_i de j n'ont pas intérêt à imiter les agents θ_{-i} de la région $-j$. Si on appelle $\left(x_i^j(\beta), z_i^j(\beta)\right)$ la combinaison consommation/revenu brut d'un agent de caractéristique (θ_i, β) vivant dans la région j, le premier ensemble de contraintes (i) s'écrit :

$$v\left(x_i^A(\beta), \frac{z_i^A(\beta)}{\theta_i}\right) \geq v\left(x_{-i}^A(\beta), \frac{z_{-i}^A\left(\widehat{\beta}\right)}{\theta_i}\right), \qquad \forall i, -i;\ \forall \beta, \widehat{\beta}, \tag{3.19}$$

$$v\left(x_i^B(\beta), \frac{z_i^B(\beta)}{\theta_i}\right) \geq v\left(x_{-i}^B(\beta), \frac{z_{-i}^B\left(\widehat{\beta}\right)}{\theta_i}\right), \qquad \forall i, -i;\ \forall \beta, \widehat{\beta}. \tag{3.20}$$

Ces contraintes correspondent aux conditions d'incitation rencontrées usuellement. Au contraire, les contraintes d'incitation (ii) sont inédites. Elles requièrent

$$v\left(x_i^A(\beta), \frac{z_i^A(\beta)}{\theta_i}\right) \geq v\left(x_{-i}^B(\beta), \frac{z_{-i}^B\left(\widehat{\beta}\right)}{\theta_i}\right), \qquad \forall i, -i;\ \forall \beta, \widehat{\beta}, \tag{3.21}$$

$$v\left(x_i^B(\beta), \frac{z_i^B(\beta)}{\theta_i}\right) \geq v\left(x_{-i}^A(\beta), \frac{z_{-i}^A\left(\widehat{\beta}\right)}{\theta_i}\right), \qquad \forall i, -i;\ \forall \beta, \widehat{\beta}, \tag{3.22}$$

et expriment donc le fait qu'*un agent ne doit pas avoir intérêt à émigrer pour imiter.* Blackorby, Brett, and Cebreiro (2007) étudient ensuites les propriétés des barèmes optimaux en A et B, que nous n'allons pas présenter ici en détail. La leçon principale est que, dans un contexte fiscal de coopération, l'émigration n'affecte pas substantiellement les propriétés qualitatives des taux marginaux d'imposition dans la mesure où la décision de localisation ne correspond pas à une décision à la marge. L'implication en termes de politiques publiques est assez claire : renforcer la coopération fiscale entre Etats limite la capacité des agents compétents les plus mobiles à échapper à un impôt trop élevé et restaure des marges de manœuvre en matière de redistribution.

3.6. Conclusion

Les différents modèles présentés permettent d'étayer l'idée que la mobilité des agents de haute compétence a un impact important sur la progressivité des barèmes fiscaux. Afin de parvenir à cette conclusion, des hypothèses souvent assez restrictives sont introduites, sur les interactions entre Etats, sur la forme de l'impôt, sur l'objectif social et sur l'absence de certaines réactions comportementales en particulier. Ceci s'explique par la difficulté du problème étudié.

Notre tour d'horizon s'achève par une série de modèles (Osmundsen, 1999; Osmundsen, Schjelderup, and Hagen, 2000) qui, sans relever véritablement de la théorie de l'imposition optimale, dégagent des pistes utiles pour la prise en compte par celle-ci de l'incidence de la mobilité des personnes talentueuses. Ces modèles appliquent la cadre d'analyse de la théorie des contrats au problème de la taxation internationale des agents mobiles. Ils s'appuient à cette fin sur Lewis and Sappington (1989) et Maggi and Rodriguez-Clare (1995) qui introduisent des contraintes de participation dépendant du type aux côtés des conditions d'incitation usuelles. Cette démarche est motivée par l'idée que les agents les plus compétents sont habituellement plus mobiles en raison d'utilités de réservation plus élevées. Le programme d'imposition optimale sur le revenu est alors formulé comme un pur problème d'extraction de rente informationnelle, les préférences individuelles retenues étant linéaires en revenu et l'objectif social utilitariste.

L'étape logique suivante consiste à intégrer les contraintes de participation dépendant du type au modèle d'imposition optimale sur le revenu, sans perdre sa spécificité qui est d'éclairer le dilemme entre équité et efficacité. Les chapitres 4 et 5 s'inscrivent dans cette perspective. Ils s'appuient à cette fin sur une version du modèle de Mirrlees où les arbitrages individuels s'opèrent en marges intensives et considèrent l'impact de l'émigration potentielle des agents sur le barème optimal linéaire, puis non-linéaire, d'un pays donné.

CHAPITRE 4

L'IMPÔT LINÉAIRE OPTIMAL LORSQUE LES AGENTS TALENTUEUX VOTENT AVEC LEURS PIEDS

4.1. INTRODUCTION[1]

About 34 000 income taxpayers leave France each year since 2000 and 70% of them settle down in Europe or in North America (DGI, 2005), mostly in countries where income taxes are lower. Since these individuals paid three times more taxes than the average taxpayer, this example suggests that international differences in taxes should be regarded as one of the determinants of their migration decision making. Such a determinant is consistent with John Hicks's idea that the migration decision is determined by a comparison of earnings opportunities across countries, net of migration costs, on which practically all modern economic studies of migration are based (Borjas, 1999; Sjaastad, 1962).

The mobility of highly productive individuals between the most developed countries raises specific issues. First, it does not only induce losses in taxes but also in productive capacity in the countries which are left by these individuals. Second, governments have few alternatives but to lower taxes, that is to reduce redistribution, to prevent the departure of high-skilled individuals. The set of instruments at the governments' disposal is thus more limited than when they face tax evasion (Chander and Wilde, 1998) : they have "carrots" but no "sticks". There is therefore a specific conflict between their desire to maintain the national income per capita in keeping taxes down and their desire to sustain the redistribution programme. Third and consequently, this mobility of individuals appears as a new constraint on the design of the optimal income tax in the developed countries.

[1] This chapter is joint work with Alain Trannoy. We are grateful to an anonymous referee and the organizers of the HECER workshop on fiscal federalism, Erkki Koskela and Panu Poutvaara. The usual caveat applies.

Our purpose is to study the impact of this specific mobility on the optimal linear tax scheme à la Mirrlees. Linear taxes have been studied by Sheshinski (1972); Atkinson (1973, 1997); Romer (1976); Dixit and Sandmo (1977); Helpman and Sadka (1978); Hellwig (1986) and Tuomala (1985) who used first-order conditions to derive formulae giving the optimal marginal rates of tax in a closed economy and studied the properties of the social optimum. Extensive numerical results have been provided by Stern (1976).

The focus on linear income tax schemes has two justifications. First, linear taxes seem to have many advantages. From the technical point of view, it is not necessary to introduce the incentive compatibility constraints in the optimal income tax problem explicitly, provided the Spence-Mirrlees condition is met. In addition, linear taxes are very simple for the taxpayers as well as for the Inland Revenue : they consist indeed of the payment of a basic income to everyone, funded by a proportional tax rate on all income. Furthermore, this simplicity could favour a decrease in the compliance costs. Second, the idea of a linear tax scheme is being actively discussed in a number of countries and has been recently implemented in practice in Estonia, Latvia, Lithuania, Russia, Ukraine, Slovakia, or Romania, among others.

The effects of individual mobility on the optimal linear income tax scheme have been studied by Wilson (1982c,a). These papers examine how potential emigration changes the optimal linear income tax. For this purpose, the methodology consists in comparing the tax parameters of an economy in which residents are free to migrate at will with those of an economy in which certain individuals may not enter or leave the country. The differences between both economies isolate the effects of potential mobility on the optimal tax scheme, at productivity levels where individuals cannot change their status between resident and emigrant. The main result is that partially closing the economy raises the optimal marginal tax and minimum income.

The present paper is devoted to optimal linear income taxation when there is a continuum of individuals who differ in productivity and migration costs and face consumption-leisure choices in the absence of unemployment. It examines how the results concerning the optimum income tax schedule in a Mirrleesian economy A have to be adapted when the agents are allowed to vote with their feet and settle down in a less redistributive country B. Since at this stage we are not interested in the reversed question, we consider that B's optimal income tax schedule is given. The only coherent theoretical case is thus that in which B is a laissez-faire country. Otherwise emigration from A to B would alter the budget balanced constraint of B's government which should therefore adapt its optimal policy. In addition, we assume that all individuals stay intially in A because we are not interested herein in emigration of low skilled individuals from B to A but in the impact of the threat of migration by highly skilled individuals on A's tax schedule.

We introduce type-dependent participation constraints in A's optimal income tax problem to model the possibility that the individuals vote with their feet. These constraints express the fact that an individual will leave A if the net utility he obtains there is less than his reservation utility, equal to the maximum utility he can obtain abroad, taking his migration cost into account. Such constraints have not been used in the previous models of optimal income taxation dealing with individual mobility. This is rather surprising if one considers that optimal income taxation is a principal-agent model with "false moral hazard" (Laffont and Martimort, 2002).

The social objective is more difficult to specify when the individuals are allowed to vote with their feet. It does not only depend on the government's aversion to income inequality, but also on the set of individuals whose welfare is to count (Mirrlees, 1982). We distinguish two different social criteria. Under the National criterion, A's government takes the welfare of its nationals into account, whilst ensuring that every national is staying in the home country. This criterion allow us to examine the impact of the threat of migration on the optimal linear tax scheme. We also consider a social criterion allowing changes in the size of A's resident population to determine if it is always socially optimal to prevent emigration of the highly productive individuals in A's resident population.

Our approach differs therefore from Wilson (1980, 1982c) in at least three ways. First, Wilson (1980, 1982c) considers that the contribution of an individual's utility to social welfare does not depend on whether or not he is a resident or an emigrant. We thus look at different social welfare criteria. Second, we introduce participation constraints to examine the effects of potential emigration and address the issue of the optimal size of the resident population. Third, migration costs are explicitly taken into account.

The basic assumptions we make are the following. First, individual productivity, equal to gross wage, does not change with the country of residence. In other words, both countries have the same production function so that our framework differs from the literature on the brain drain in which the key parameter is the difference in productivity (Bhagwati and Wilson, 1989). Second, migration costs depend on each individual's productivity. Productivity is thus the sole parameter of heterogeneity in the population. Third, migration costs are monotonic (increasing, constant or decreasing) but do not increase faster than the laissez-faire utility. It should be noted that we place no restrictions on their level. On the contrary, if individuals were initially living in B, we would have to introduce an additional assumption on the level of the migration costs in order to prevent emigration of low-skilled workers from B to A in which we are not interested presently.

The main results of the paper provide an argument against using linear income taxes when highly skilled individuals are potentially mobile for tax purposes. It does not mean that linear taxes should not be used in the presence of migration, but that the cost-benefit analysis of the restriction to linearity should take this point into account. We first use the National criterion to look at the impact of the threat of migration of highly skilled individuals on the optimum linear tax scheme A's government should implement. The fact that participation constraints have to be satisfied for the most demanding individuals prevents the government from reducing the utility of the other highly productive individuals to their reservation utility. These individuals are thus left with a rent. Consequently, redistribution of income within the population has to be reduced. We provide an illustration on French data to assess the loss in social welfare resulting from the openness of the economy. The magnitude of this loss emphasizes that linear taxes are in fact lacking in degrees of freedom when many constraints have to be taken into account. Since the satisfaction of the participation constraints for the most demanding individuals living in A seems to constrain the social optimum very much, we employ the Resident criterion to examine the consequences of allowing emigration of the highest skilled individuals in A's resident population. It appears that the social welfare is not monotonic in the upper productivity level in A. In particular, numerical results show that letting national income per capita to decrease due to emigration may be a lesser evil than reducing income redistribution sufficiently to prevent the emigration of those with highest income. There is therefore a trade-off between redistribution and population size.

The paper is organized as follows. Section 2 presents the model. Section 3 examines the properties of the optimal tax scheme in A when A's government aims at preventing emigration of its nationals. Section 4 provides numerical results concerning the French economy. Section 5 concludes.

4.2. THE MODEL

The world consists of two countries, A and B. The instruments at the disposal of A's government are a minimum income m ($m \in R$) and a constant tax rate t ($0 \leq t \leq 1$) levied on earnings z. The restriction to marginal tax rates which are not greater than 1 is required for the incentive compatibility constraints to be satisfied, as stressed below. The restriction to non-negative marginal tax rates is justified herein by our focus on purely redistributive tax policies. The linear income tax function in A is thus

$$T(z) = tz - m. \tag{4.1}$$

B is committed to being a laissez-faire country. Every individual is initially living in A.

4.2.1. Population

The individuals differ in productivity θ. The distribution function of θ, denoted F, is defined on a closed interval $\left[0, \overline{\theta}\right] \subseteq \mathbb{R}^{+}$ where it admits a density function $f > 0$. This distribution is common knowledge, but individual productivity is private information.

Productivity levels, and therefore wages in the absence of taxation, are independent of the country in which the individuals are working.

4.2.2. Individual Behaviour

All individuals have the same preferences over consumption x and labour l, represented by a strictly concave utility function $U : R^{+} \times \left[0, \overline{l}\right] \rightarrow R$. The following assumptions are made.

Assumption 4.1 *U is a $\mathcal{C}^2$ strictly concave function such that $U_x > 0$, $U_l < 0$ and $U \rightarrow -\infty$ as $x \overset{>}{\rightarrow} 0$ or $l \overset{<}{\rightarrow} \overline{l}$.*

Assumption 4.2 *Leisure is a normal good.*

Gross income z is given by $z := \theta l$. Each individual decides about the optimal amount of consumption and labour so as to maximize his utility subject to his budget constraint. The individual budget constraint in A reads

$$x = z - T(z) = (1-t)z + m. \tag{4.2}$$

The individual budget constraint in B is simply

$$x = z. \tag{4.3}$$

An individual working in A maximizes U subject to (4.2). The corresponding first-order condition, $\theta(1-t)U_x' + U_l' = 0$ or $l = 0$ and $\theta(1-t)U_x' + U_l' < 0$, defines implicitly the Marshallian labour supply l_A and the consumption function x_A in A,

$$l_A := l_A(\theta; t, m) \text{ and } x_A := \theta(1-t)l_A(\theta; t, m) + m. \tag{4.4}$$

The gross income function in A is thus $z_A := \theta l_A(\theta; m, t)$. Substituting x_A and l_A in U, one obtains the indirect utility in A,

$$V_A(\theta; t, m) := U(x_A, l_A). \tag{4.5}$$

By the envelope theorem, one gets

$$V_A'(\theta) = U_x.(1-t)l_A \tag{4.6}$$

and

$$\partial V_A / \partial m = U_x > 0. \tag{4.7}$$

Hence,

$$\frac{\partial V_A}{\partial t} = -\theta l_A U_x = -\theta l_A \frac{\partial V_A}{\partial m} \leq 0. \tag{4.8}$$

The marginal rate of substitution between income and consumption $s(x_A, z_A; \theta)$ in A is equal to

$$s(x_A, z_A; \theta) := -\frac{U_l}{\theta U_x}. \tag{4.9}$$

An individual working in B maximizes U subject to (4.3). His consumption function and labour supply are $x_B = x_B(\theta)$ and $l_B = l_B(\theta)$. By the envelope theorem, the indirect utility in B defined by

$$V_B(\theta) := U(x_B, l_B) \tag{4.10}$$

is strictly increasing in θ.

4.2.3. Emigration and Participation Constraints

An individual who decides to leave A has to pay a strictly positive *migration cost*, denoted c. This cost is introduced in the model as a *time-equivalent* loss in utility and corresponds to different material and psychic costs of moving : transportation of persons and household's goods, forgone earnings, costs of speaking a different language and adapting to another culture, costs of leaving his family and friends, etc. If "these costs probably vary among persons, the sign of the correlation between costs and wages is ambiguous" (Borjas, 1999, p. 12). We assume herein that migration costs depend on productivity, i.e. $c : \left[0, \overline{\theta}\right] \rightarrow R^{++}$, and that only their distribution is known to A's government. In addition,

Assumption 4.3 *Migration costs c, monotonic and twice continuously differentiable, do not increase faster than the laissez-faire utility.*

In this setting, A's government knows c if it knows θ. Assumption 4.3 concerns the *rate of increase* of the migration cost function c and *no* assumption is made on the level of c. Migration costs are allowed to be constant, decreasing, or increasing provided $c'(\theta) < V_B'(\theta)$.

The *reservation utility*, defined as the maximum utility an individual living and working in A can obtain abroad, is given by $V_B - c$. Assumption 4.3 amounts therefore to considering that outside opportunities are increasing in productivity. An individual will leave A if and only if his utility in A is less than his reservation utility. Consequently, the *participation constraint* for the θ-individuals is defined as

$$V_A(\theta; t, m) \geq V_B(\theta) - c(\theta). \tag{PC}$$

4.2.4. Social Objective and Tax Policy

We define a *national* as an individual born in A. Hence, all individuals have A's nationality. Some of these individuals may choose to vote with their feet and settle down in B. We consider that there is a partition of the population, with the low-skilled being in A, because the focus is on high-skilled emigration. In addition, since emigration of a set of measure zero does not capture any economic intuition, A's resident population must be compact. This is summarized as follows.

Assumption 4.4 *A's resident population is an interval of types* $\left[0, \widehat{\theta}\right]$, *with* $\widehat{\theta} \in \left[0, \overline{\theta}\right]$.

A's government intends to implement the income tax policy corresponding to the best compromise between its desire to redistribute income and its concern about limiting the disincentive effects of the tax system. It seems sensible to consider that it is not able to levy taxes in B since the fiscal prerogative is closely linked to national sovereignty and that it is not willing to redistribute income to the individuals in B. Focusing on purely redistributive tax schemes, the tax revenue constraint of A's government is

$$t\int_0^{\widehat{\theta}} \theta l_A(\theta;t,m)\,dF(\theta) \geq mF\left(\widehat{\theta}\right). \tag{TR}$$

The desire to redistribute income is captured through the government's aversion to income inequality $\rho \geq 0$. A zero aversion corresponds to a Utilitarian government and an infinite aversion to a Rawlsian one (maximin).

Under the *National* criterion, A's government aims at maximizing the average social welfare of its nationals whilst maintaining the national productive capacity in preventing emigration of its nationals. The social objective reads therefore

$$W_{A,\rho}^N(t,m) = \frac{1}{F\left(\widehat{\theta}\right)}\int_0^{\widehat{\theta}} \phi_\rho\left(V_A(\theta;t,m)\right)dF(\theta) \text{ with } \widehat{\theta} = \overline{\theta}. \tag{4.11}$$

The function ϕ is defined as $\phi_{\rho\neq 1}(V_A) = V_A^{1-\rho}/(1-\rho)$ for $\rho \neq 1$ and $\phi_1(V_A) = \ln V_A$ for $\rho = 1$. This criterion corresponds to the idea, formulated by Jean Bodin, that "the only source of welfare is mankind itself" (Bodin, 1578).

We also consider a *Resident* criterion allowing emigration of the highest productive individuals. A government adopting this criterion aims at maximizing the average social welfare of its residents[2]. The social objective is thus

$$W_{A,\rho}^R\left(t,m;\widehat{\theta}\right) = \frac{1}{F\left(\widehat{\theta}\right)}\int_0^{\widehat{\theta}} \phi_\rho\left(V_A(\theta;t,m)\right)dF(\theta). \tag{4.12}$$

The basic idea is that an economic policy should take the welfare of the taxpayers into account. Since the welfare of the nationals living in B does not count, A's government has to choose the optimal size of its population ; it thus faces "*different* number choices" (Parfit, 1984).

Participation constraints (PC) are thus taken into account for every nationaland does not care for its nationals if they are living in B. The participation constraints read

$$V_A(\theta;t,m) \geq V_B(\theta) - c(\theta) \text{ for all } \theta \leq \widehat{\theta}. \tag{4.13}$$

The Spence-Mirrlees condition is assumed to be met.

Assumption 4.5 *$s(x,z;\theta)$ is strictly decreasing in θ.*

[2] This kind of social welfare functional is known to violate the so-called "Mere Addition Paradox" : the addition of individuals whose utility is less than the average utility in the initial population is regarded as suboptimal even if this change in population size affect no one else and does not involve social injustice. However, since we are focusing on emigration of individuals whose utility is greater than the average utility in A, such Paradox cannot occur.

Under Assumption 4.5, $z_A' \geq 0$ and then the incentive compatibility constraints are satisfied (see Hellwig (1986)). We also note that, by (4.6), the restriction $t \leq 1$ is required to ensure that $V_A'(\theta) = U_x.(1-t)\, l_A$ is non-decreasing in θ. In the rest of the paper, we exclude the case where t is equal to 1, which implies a zero production in A.

Consequently, the linear income tax problems we focus on may be written as

Problem 4.1 (National Criterion) $W_{A,\rho}^{M} = \max_{t,m} W_{A,\rho}^{M}(t,m)$ *subject to* $\widehat{\theta} = \overline{\theta}$, *(TR), and*

$$V_A(\theta; t, m) \geq V_B(\theta) - c(\theta) \ \ \text{for all } \theta \leq \widehat{\theta}. \tag{4.14}$$

Problem 4.2 (Resident Criterion) $W_{A,\rho}^{R}\left(\widehat{\theta}^{*}\right) = \max_{t,m,\widehat{\theta}} W_{A,\rho}^{R}\left(t, m; \widehat{\theta}\right)$ *subject to (TR) and (4.14).*

4.3. NATIONAL CRITERION

This section examines how the threat of migration for tax purposes alters the optimum linear tax scheme in A when A's government adopts the National criterion. In this case, the optimum tax scheme is solution to Problem 4.1.

4.3.1. The Tax Possibility Set

We begin by expressing the participation constraints as a restriction on the feasible (t,m)-pairs. For this purpose, we define $m^{PC}(\theta; t, V_B - c)$ as the minimum lump-sum element m that is required for a θ-individual to obtain his reservation utility $V_B(\theta) - c(\theta)$ in A when the marginal tax rate is t, namely

$$m^{PC}(\theta; t, V_B - c) = \min\left\{m \in R : U\left(\theta(1-t)\, l_A(\theta; t, m) + m, l_A(\theta; t, m)\right) \geq V_B(\theta) - c(\theta)\right\}. \tag{4.15}$$

$m^{PC}(\theta; t, V_B - c)$ always exists under our assumptions. Since (PC) must be satisfied for *all* individuals, the minimum income m must be chosen so that

$$m \geq m^{PC}(t; V_B - c) := \max_{\theta \in \left[0, \overline{\theta}\right]} m^{PC}(\theta; t, V_B - c). \tag{4.16}$$

The upper graph of $m^{PC}(t; V_B - c)$, denoted $\Pi^{PC}(V_B - c)$, is the set of (t,m)-pairs which are compatible with the participation constraints (PC). Since $c(\theta) > 0$, it is obvious that the laissez-faire allocation $(t,m) = (0,0)$ belongs to Π^{PC}.

Under Assumption 4.2, $\int_0^{\overline{\theta}} \theta l_A(\theta; t, m)\, dF(\theta) - m$ is continuously decreasing in m and bounded below $(l_A \geq 0)$ for any given $0 \leq t \leq 1$.There is thus a unique m such that (TR) is active for a given t. The tax revenue constraint (TR) defines therefore a continuous function $t \to m^{TR}(t)$ on $0 \leq t \leq 1$, which is the Laffer curve. We define Π^{TR} as its lower graph and note that the laissez-faire allocation $(t,m) = (0,0)$ belongs to this set.

The tax possibility set is defined by

$$\Pi\left(V_B - c\right) := \Pi^{PC}\left(V_B - c\right) \cap \Pi^{TR}. \tag{4.17}$$

The laissez-faire allocation, which belongs to this set, is always feasible.

4.3.2. The Social Indifference Curves

The social indifference curves capture equity. For any given level of social welfare $\overline{W}$, they have equation

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi_\rho\left(V_A\left(\theta; t, m\right)\right) dF\left(\theta\right) - \overline{W} = 0 \tag{4.18}$$

in the (t, m)-space. We note that

$$\frac{\partial}{\partial t}\int_0^{\overline{\theta}} \phi_\rho\left(V_A\right) dF\left(\theta\right) = -\int_0^{\overline{\theta}} \phi_\rho'\left(V_A\right)\left(\theta l_A \frac{\partial V_A}{\partial m}\right) dF\left(\theta\right) \leq 0 \tag{4.19}$$

$$\frac{\partial}{\partial m}\int_0^{\overline{\theta}} \phi_\rho\left(V_A\right) dF\left(\theta\right) = \int_0^{\overline{\theta}} \phi_\rho'\left(V_A\right) \frac{\partial V_A}{\partial m} dF\left(\theta\right) > 0, \tag{4.20}$$

because of Eq. (4.8) and (4.7). As a result, the social indifference curves slope up for any finite aversion to income inequality ρ. In the Rawlsian case, A's government aims at maximizing the lump-sum tax m received by the worst-off members in A population. The social indifference curves are therefore horizontal straight lines in the (t, m)-space.

4.3.3. The Properties of the Solution

We start with the following obvious property.

Property 4.1 *The tax revenue constraint (TR) is always active at the social optimum.*

Indeed, let us assume that there is a social optimum such that (PC) is binding at some θ^* whilst (TR) is inactive. We thus have

$$\delta = t\int_0^{\overline{\theta}} \theta l_A\left(\theta, t, m^{PC}\left(\theta^*, t, V_B - c\right)\right) dF\left(\theta\right) - m^{PC}\left(\theta^*, t, V_B - c\right) > 0. \tag{4.21}$$

Since it is possible to share out δ among the individuals while increasing their indirect utility, one obtains a contradiction.

Property 4.1 implies that there are *two types of social optima* when the individuals can vote with their feet. Either the participation constraints (PC) are inactive for all $\theta \in \left[0, \overline{\theta}\right]$ and we are back to the closed economy optimum, or there is at least one θ for which (PC) is active. In the latter case, the social optimum is at the junction of the graphs of $m^{PC}\left(t; V_B - c\right)$ and $m^{TR}\left(t\right)$;

as a result, we cannot rely anymore on the standard first-order conditions to characterize the optimum and derive a marginal tax rate formula.

Eq. (4.16) is equivalent to the satisfaction of (PC) for all $\theta \in \left[0, \overline{\theta}\right]$. The participation constraints are thus binding at productivity levels θ^* defined as

$$\theta^* \in \arg \max_{\theta \in \left[0, \overline{\theta}\right]} m^{PC}\left(\theta; t, V_B - c\right). \tag{4.22}$$

One thus obtains the following property.

Property 4.2 *Let $m^{PC}\left(\theta; t, V_B - c\right)$ be strictly monotonic in θ. Then, there is at most one θ^* where the participation constraints (PC) are active. When $m^{PC}\left(\theta; t, V_B - c\right)$ is strictly increasing in θ, θ^* is $\overline{\theta}$.*

When $m = m^{PC}\left(\theta; t, V_B - c\right)$, we have by definition of $m^{PC}\left(\theta; t, V_B - c\right)$,

$$U\left(\theta\left(1-t\right) l_A\left(\theta; t, m\right) + m, l_A\left(\theta; t, m\right)\right) = V_B\left(\theta\right) - c\left(\theta\right). \tag{4.23}$$

Differentiation of this relation yields

$$\begin{aligned} \frac{\partial m^{PC}\left(\theta; t, V_B - c\right)}{\partial \theta} &= \frac{V_B'\left(\theta\right) - c'\left(\theta\right)}{U_x} - \left(1-t\right) l_A\left(\theta; t, m\right) \\ &= \frac{1}{U_x}\left[\frac{dV_A\left(\theta; 0, 0\right)}{d\theta} - c'\left(\theta\right) - \frac{dV_A\left(\theta; t, m\right)}{d\theta}\right], \end{aligned} \tag{4.24}$$

where (4.6) has been used to obtain the latter equality. We note that, by (4.8),

$$\frac{\partial m^{PC}\left(\theta; t, V_B - c\right)}{\partial \theta} = \frac{1}{U_x}\left[V_B'\left(\theta\right) - c'\left(\theta\right) + \frac{1-t}{\theta}\frac{\partial V_A}{\partial t}\right]. \tag{4.25}$$

Since $V_B'\left(\theta\right) - c'\left(\theta\right) > 0$ under Assumption 4.3 and $\partial V_A / \partial t \leq 0$ by (4.8), the sign of $\partial m^{PC}\left(\theta; t, V_B - c\right) / \partial \theta$ is ambiguous in the general case.

To have further insight into the monotonicity of $m^{PC}\left(\theta; t, V_B - c\right)$, we look at preferences which are quasilinear in consumption,

$$U = x - v\left(l\right), \text{ with } v', v'' > 0. \tag{4.26}$$

The labour supply is thus equal to

$$l_A\left(\theta; t, m\right) = v'^{-1}\left(\theta\left(1-t\right)\right). \tag{4.27}$$

Since $U_x = 1$, (4.6) yields

$$\frac{dV_A\left(\theta; 0, 0\right)}{d\theta} - \frac{dV_A\left(\theta; t, m\right)}{d\theta} = v'^{-1}\left(\theta\right) - \left(1-t\right) v'^{-1}\left(\theta\left(1-t\right)\right). \tag{4.28}$$

The function $v'^{-1}(.)$ is strictly increasing since, by assumption, $v' > 0$. One thus gets

$$(1-t)\, v'^{-1}(\theta(1-t)) < v'^{-1}(\theta(1-t)) < v'^{-1}(\theta) \qquad (4.29)$$

provided $0 < t < 1$. By (4.24), $m^{PC}(\theta; t, V_B - c)$ is strictly increasing in θ for all $0 < t < 1$ when $c'(\theta) \leq 0$ and preferences are quasilinear in consumption. The following property is obtained.

Property 4.3 *Let preferences be quasilinear in consumption and migration costs be non-increasing. Then, θ^* is $\overline{\theta}$.*

4.4. ILLUSTRATION ON FRENCH DATA

This section provides numerical simulations of the optimal linear income tax the calibration of which is chosen to roughly describe the French economy. A linear income tax has not been yet adopted in France but there is a trend towards a reduction in tax rate brackets, from 13 in 1982 to 5 in 2007. In addition, a recent report by Le Cacheux and Saint-Etienne (2005) has proposed an income tax levied in three brackets. We focus on the impact of the threat of migration when the government adopts the National criterion, before looking at the Resident criterion which allows emigration of the most productive individuals living in A.

4.4.1. Calibration

The government's aversion to income inequality reflects a political choice. We will consider the utilitarian and the Rawlsian cases which are sufficiently manageable to compute the social indifference curves in the (t, m)-space.

It is usual, in optimal taxation, to describe the distribution of skills within the population with a lognormal (or a Pareto) distribution. We use the work of Laslier, Trannoy, and Van Der Straeten (2003, Appendix C) to obtain a Kernel estimation of the distribution of skills in France[3], based on the data from the survey "Budget des familles", year 1995. The median individuals have productivity equal to 13 320 euros. We obtain a lognormal distribution with mean 0.2398 and variance 0.4403. Since more than 99.99% of the population has a productivity level which is less than 5 times that of the median individuals, we set the upper productivity level to 66 600 euros and distribute the remaining population according to the same lognormal distribution between 0 and 66 600 euros.

Following d'Autume (2000) who provides simulations for the optimum tax schedule in France in the absence of individual mobility, we concentrate on the special case where there are no income effects on labour supply and the elasticity of labour supply with respect to the net-of-tax

[3]Since our model does not take the family size into account, the population is restricted to single individuals living in France.

wage rate is constant. If e denotes this elasticity, the utility function is given by

$$U(x,l) = x - \frac{l^{1+1/e}}{1+1/e}. \tag{4.30}$$

It seems sensible to choose a benchmark value of $e = 0.2$. Using (4.30), one gets

$$\begin{cases} V_A(\theta;t,m) = \dfrac{\theta^{1+e}(1-t)^{1+e}}{1+e} + m, \\ V_B(\theta) - c(\theta) = \dfrac{\theta^{1+e}}{1+e} - c(\theta). \end{cases} \tag{4.31}$$

Migration costs are the new ingredient of our model and play therefore a large part in the determination of the optimal income tax scheme. Since our model is static, these "time-equivalent" costs as well as the utility levels should be regarded as expected values. Migration costs amount to all the costs an individual will have to pay because of his choice of migration. If these costs are used in the standard economic model of migration (see Borjas (1999)), there are however few empirical work concerning their level[4]. Since psychological costs are very difficult to estimate, we propose to focus on material migration costs (transportation of persons and goods, costs of visiting the previous home country from time to time to meet family and friends, etc.). As they do not vary significantly from person to person, we concentrate on constant migration costs in the rest of the paper ($c(\theta) = c$). In the simulation results, we consider that migration costs are paid once a year. We choose a benchmark value of 9 000 euros per annum, which corresponds to the costs of migration to Australia a single person faces, and provide sensitivity analysis. Utility levels and social welfare are expressed in euros per year.

By (4.31), the participation constraint at θ is equivalent to

$$V_A(\theta;t,m) \geq V_B(\theta) - c \Leftrightarrow m \geq \frac{\theta^{1+e}}{1+e}\left[1-(1-t)^{1+e}\right] - c = m^{PC}(\theta,t;V_B - c). \tag{4.32}$$

Since $m^{PC}(\theta,t;c)$ is strictly increasing in θ for $0 < t < 1$, the minimum m required to satisfy all participation constraints (PC) given $V_B - c$ is

$$m^{PC}(t;V_B - c) = \frac{\widehat{\theta}^{1+e}}{1+e}\left[1-(1-t)^{1+e}\right] - c. \tag{4.33}$$

Given t, the maximum m compatible with the tax revenue constraint (TR) is

$$m^{TR}(t) = \frac{t(1-t)^e}{F\left(\widehat{\theta}\right)} \int_0^{\widehat{\theta}} \theta^{1+e} f(\theta)\, d\theta. \tag{4.34}$$

[4]The IZA Database for Migration Literature (http ://www.iza.org/iza/en/webcontent/links/migration) provides 34 matches for "moving costs". These references are mainly theoretical or estimate the macroeconomic costs of migration.

Social Welfare	Closed Eco	Open Eco	Loss in Welfare
Non Linear Tax	13089	12566	-4.0%
Linear Tax	11846	2493	-79.0%
Loss in Welfare	-9.5%	-80.2%	-

TAB. 4.1 – Linear vs non-linear optimal tax schemes in our French economy (National criterion, Rawls,c=9000 euros/year)

The tax possibility set of our French economy is thus given by

$$\Pi\left(V_B - c\right) = \left\{(t, m) : m^{PC}\left(t; c\right) \leq m \leq m^{TR}\left(t\right)\right\}. \tag{4.35}$$

The social indifference curve corresponding to the social welfare level $\overline{W}$ in the (t, m)-space has equation

$$m\left(t\right) = \overline{W} - \frac{1}{F\left(\widehat{\theta}\right)} \frac{\left(1-t\right)^{1+e}}{1+e} \int_0^{\widehat{\theta}} \theta^{1+e} f\left(\theta\right) d\theta, \tag{4.36}$$

when the government of A is Utilitarian and $m = \overline{W}$ when it is Rawlsian.

4.4.2. National Criterion

Under the National criterion, the solution to the optimal linear income tax is a corner solution given by Properties 4.1 and 4.3.

In Figure 4.1, we have used (4.33), (4.34) and (4.36) to obtain the social optimum when migration costs are equal to 9 000 euros per year. $\Pi^{PC}\left(t, V_B - c\right)$ is the set of (t, m)-pairs above $m^{PC}\left(t, V_B - c\right)$, whilst $\Pi^{TR}\left(t\right)$ is the set of (t, m) below the Laffer curve $m^{TR}\left(t\right)$. The tax possibility set of the economy is the junction of both sets and is therefore significantly restricted compared with the one obtained in closed economy ($\Pi^{TR}\left(t\right)$). The social indifference curves associated with the highest feasible social welfare are drawn in the Utilitarian (SIC* $\rho = 0$) and Rawlsian (SIC* $\rho = \infty$) cases. The Utilitarian optimum (U^*) is the same as in closed economy and corresponds to the laissez-faire. The Rawlsian optimum (R^*) differs from the optimum which would be obtained in closed economy (R^{Closed}).

When A's government is Rawlsian, there is a decrease in social welfare by 79% because of potential individual mobility. The indirect social welfare, equal to the minimum income m, amounts to 2 493 euros per year, versus 11 846 euros if the economy were closed. Table 4.1 contrasts these figures with the ones obtained when A's government is not restricted to setting linear tax schemes. The loss in social welfare at the optimum due to the restriction to linear taxes is striking. Computational methods used are presented in the Appendix.

In fact, by Properties 4.1 and 4.2, both instruments of the linear tax policy have to be adjusted so that the participation constraints are satisfied for the most productive $\overline{\theta}$-individuals and the tax revenue constraint is binding. Since the participation constraint has to be binding at $\overline{\theta}$, the highly productive individuals with $\theta < \overline{\theta}$ receive a utility greater than their reservation utility

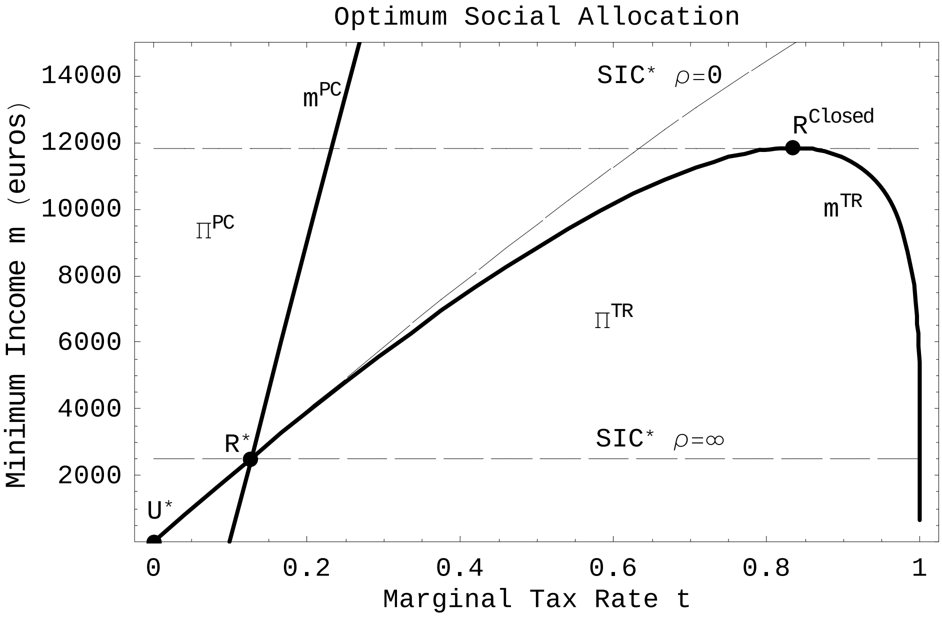

FIG. 4.1 – Social optimum allocation in our French economy when $c(\theta) = 9\,000$ euros/year

and are therefore left with a rent for staying in A (*cf.* Figure 4.2). This overcompensation explains why the redistribution in favour of the less productive individuals is hugely reduced.

The magnitude of the reduction in minimum income, that is in social welfare, depends on the cost of migration (*cf.* Figure 4.3 and Table 4.2). The higher the migration cost, the lower the reservation utility at $\overline{\theta}$ and thus the lower the rent left to the highly skilled workers. When the migration cost becomes sufficiently high, the participation constraint does no longer be active at $\overline{\theta}$ so that the difference in social welfare between the linear and non-linear solutions is the same as in the closed economy framework[5].

In conclusion, these simulations confirm the theoretical properties detected in the last section. Linear taxes seem to lack degrees of freedom when the government adopts the National criterion.

4.4.3. The Trade-Off between Redistribution and Population Size under the Resident Criterion

We now examine if preventing emigration of all high-skilled individuals is socially optimal. Indeed, the presence in A of individuals with high outside opportunities seems to constrain the

[5] In Figure 4.3 and in the linear case, the elasticity of social welfare with respect to migration costs is first almost constant, and then no longer so. Since θ^* is $\overline{\theta}$ by Property 4.3, (4.33) and (4.34) show that an increase in migration costs results in $m^{PC}(t; V_B - c)$ going down while $m^{TR}(t)$ is unaltered. Note that, in Figure 1, $m^{TR}(t)$ is almost a straight line for low marginal tax rates. Consider a slight increase in c, say at $c = 9000$. We know that the new corner solution is on $m^{TR}(t)$ and that social welfare amounts to $m^{PC}(t; V_B - c)$ since the objective is Rawlsian. Hence, the elasticity of social welfare with respect to migration costs is almost constant. This is no longer the case when $m^{PC}(t; V_B - c)$ ceases to be almost linear.

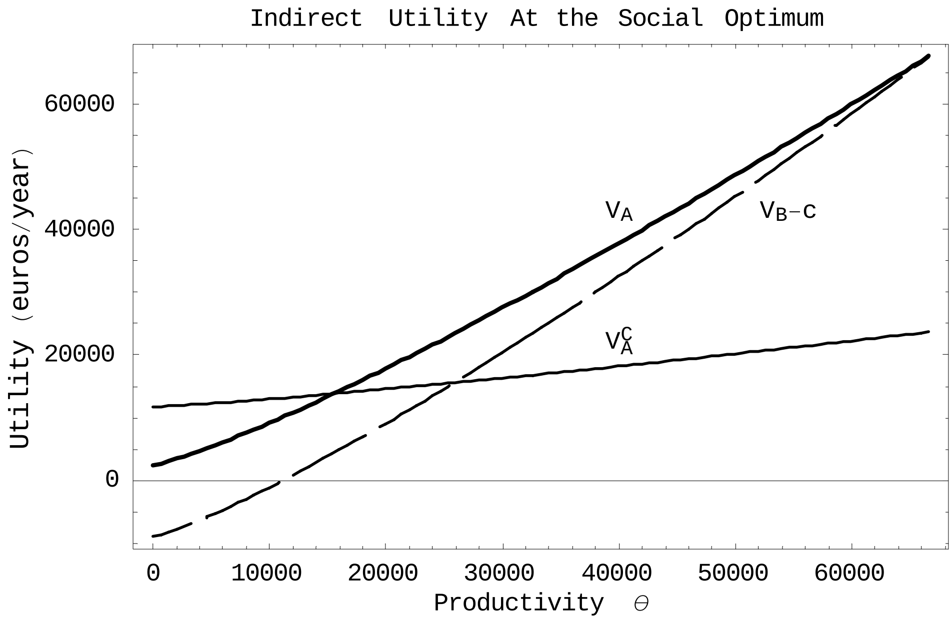

FIG. 4.2 – Indirect utility V_A, reservation utility $V_B - c$ and indirect utility if the economy were closed V_A^C (National criterion, parameters of our French economy, $c(\theta) = 9000$ euros/year)

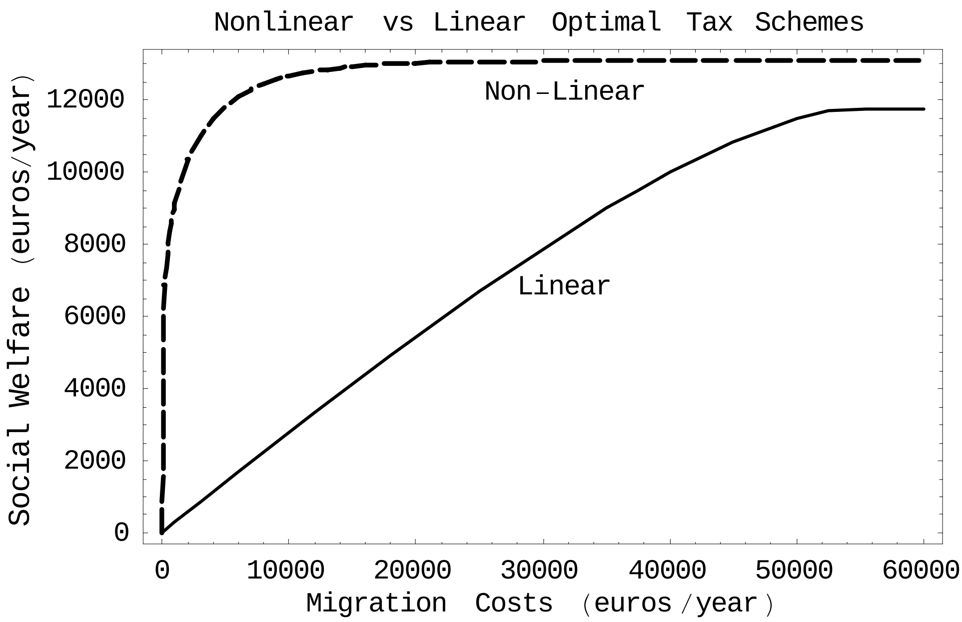

FIG. 4.3 – Indirect social welfare with respect to migration costs for a linear tax scheme (solid line) and a nonlinear tax scheme (dashed line). (National criterion)

	Closed Eco	With Potential Mobility								
c	-	0	3000	6000	9000	12000	18000	25000	35000	55500
t	83.3%	0	4.2%	8.4%	12.6%	17.0%	25.6%	35.9%	51.0%	83.3%
m	11846	0	848	1688	2493	3330	4915	6683	8991	11846
$W_{A,\infty}^{M}$	11846	0	848	1688	2493	3330	4915	6663	8991	11846

$\overline{c}$, m and $W_{A,\infty}^{M}$ are expressed in euros/year

TAB. 4.2 – Optimal tax scheme in our French economy when c=9000 euros/year

National optimal tax scheme significantly. We thus study how a change in the supremum of A's resident population, $\widehat{\theta}$, alters the tax possibility set and the social welfare under the Resident criterion.

The Changes in the Tax Possibility Set

The changes in the tax possibility set are obtained by differentiation of (4.33) and (4.34),

$$\frac{\partial m^{PC}\left(t;V_B-c\right)}{\partial\widehat{\theta}} = \widehat{\theta}^{e}\left[1-(1-t)^{1+e}\right]>0 \tag{4.37}$$

$$\frac{\partial m^{TR}\left(t\right)}{\partial\widehat{\theta}} = t\left(1-t\right)^{e}\frac{f\left(\widehat{\theta}\right)}{F\left(\widehat{\theta}\right)}\left[\widehat{\theta}^{1+e}-\frac{1}{F\left(\widehat{\theta}\right)}\int_0^{\widehat{\theta}}\theta^{1+e}dF\left(\theta\right)\right]>0, \tag{4.38}$$

for all $0<t<1$. A reduction in $\widehat{\theta}$ results in a decrease in $m^{PC}\left(t;V_B-c\right)$ which enlarges the set of tax schedules compatible with the participation constraints (PC), as well as in a decrease in $m^{TR}\left(t\right)$, which reduces the set of tax schedules compatible with the tax revenue constraint (TR).

The Optimal Size of A's Resident Population

For simplicity, we focus on the Rawlsian case of our French economy to examine the trade-off between redistribution and population size.

We begin by noting that if the individuals had not the possibility to threat to emigrate, it would always be socially optimal to add more productive individuals to A's resident population. Indeed, when no participation constraints have to be taken into account, adding individuals to A's resident population slackens the tax revenue constraint (TR), resulting in an increase in minimum income and social welfare. In other words, the social welfare obtained in closed economy when $\widehat{\theta}$ is the supremum of the resident population,

$$W_{A,\rho}^{C}\left(\widehat{\theta}\right):=\max_{t,m}W_{A,\rho}^{R}\left(t,m;\widehat{\theta}\right)\text{ s.t. (TR)}, \tag{4.39}$$

is monotonically increasing in $\widehat{\theta}$. An increase in population size is thus welfare-improving.

Figure 4.4 contrasts $W_{A,\infty}^{C}\left(\widehat{\theta}\right)$ with the social welfare $W_{A,\rho}^{R}\left(\widehat{\theta}\right)$ obtained under the Resident

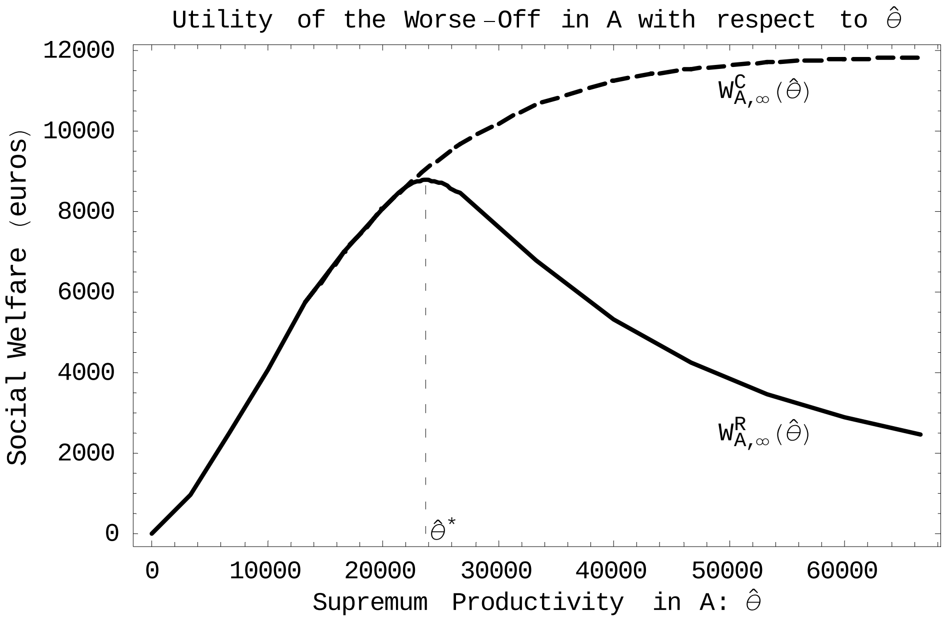

FIG. 4.4 – Indirect Social Welfare in A with respect to $\widehat{\theta}$. (Rawls, Parameters of our French Economy, $c(\theta) = 9000$ euros/year)

criterion when the individuals with productivity greater than $\widehat{\theta}$ are allowed to emigrate. The parameters are those of our French economy with a constant migration cost of 9 000 euros per year. Three points are worth noting.

First, the social welfare is not monotonic in $\widehat{\theta}$ when individuals are allowed to vote with their feet. The maximum social welfare is not obtained when the government prevents emigration of the most productive individuals who are initially living in A. Consequently, the National point of view is not the best in term of social welfare.

Second, it is socially optimal to reduce $\widehat{\theta}$ up to $\widehat{\theta}^* \simeq 23\,710$ euros. The minimum income amounts then to $\simeq 8\,795$ euros/year. The openness of the economy results therefore in a decrease in social welfare by 25.8%, versus 79% under the National criterion. In addition, the reduction in minimum income with respect to the National case with non-linear taxes amounts to 27.3%.

Third, A's optimum resident population corresponds to $\simeq$ 77.9% of its initial population. Implementing a more redistributive tax schedule, despite this inducing the departure of the upper 22.1% of the national population, increases social welfare. That corresponds to the emigration of more than 13 million people from France. The magnitude of this figure depends obviously on some simplifying assumptions we made. It would be reduced if B were more redistributive or if different types of labour were complement. It should therefore be regarded as merely illustrative of the trade-off between redistribution and population size that may occur when income taxes are linear.

4.5. CONCLUSION

The mobility of highly skilled individuals for tax purposes adds a specific conflict between the government's desire to maintain the national income per capita by keeping taxes down and its desire to sustain the redistribution programme, to the basic trade-off between equity and efficiency formulated by Mirrlees (1971). Our aim has been to examine the impact of this mobility on the optimal linear tax scheme that should be implemented in a Mirrleesian economy A.

When A's government adopts the National criterion and that the social optimum is not the same as in closed economy, the optimal linear tax scheme is a corner solution. The choice of A's government is thus very limited. In addition, the satisfaction of the participation constraints for the most demanding individuals is only possible if a rent is left to the other highly skilled individuals. This is detrimental to redistribution within A's resident population. Numerical simulations on French data have shown than the loss in welfare resulting from the restriction to linearity of the tax scheme can be substantial. Linear income taxes seems to lack degrees of freedom when the government wishes to prevent a reduction in the national income per capita. This lack might be even more acute if migration costs would depend on more than one parameter.

The study of the Resident point of view has revealed that there is a trade-off between redistribution and population size. In particular, we have given an example in which the social welfare function is not monotonically increasing with respect to the upper productivity level in A's resident population. That means that implementing a linear income tax scheme which results in the emigration of the most productive individuals initially living in A may be socially optimal. Emigration in not a source of gains in itself ; it is rather a negative but still tolerable effect of the optimal policy. Our simulations on French data depend on substantial assumptions. They are however illustrative. In particular, the magnitude of the optimal emigration we found stresses that the effects of the trade-off between the desire to redistribute income and the wish to maintain the national income per capita is not insignificant.

The advantages of linear income taxes are often emphasized. In a nutshell, it is the simplest income tax scheme that can be implemented. It has been advocated that it should decrease the compliance costs of the income tax that are significant since they would amount to $75 billions in the United States for instance (Slemrod, 1995).

Our main findings may be regarded as an argument *against* the restriction to linearity of the tax scheme. Jan Tinbergen taught us that for economic policy to work, there needs to be at least as many policy instruments as there are policy goals. When taxes have to be linear, there are only two instruments at the government's disposal. The tax policy is therefore insufficiently flexible to face a large number of constraints. The impact of participation constraints on the optimal non-linear income taxes is addressed in Simula and Trannoy (2006a) and Simula and Trannoy (2006d).

4.6. APPENDIX

We first present the methodology used to compute the optimum Rawlsian allocations for a given value of $\widehat{\theta}$ ($\widehat{\theta} = \overline{\theta}$ under the National criterion).

When the government is restricted to setting linear income taxes, the optimum tax schedule is obtained as follows. (1.) We use $m^{PC}(t; V_B - c)$ and $m^{TR}(t)$ to determine the tax possibility set $\Pi(V_B - c)$. (2.) We compute the optimal tax schedule which would be implemented in closed economy. It is solution to $\arg\max_{0 \leq t \leq 1} m^{TR}(t)$. (3.) If this allocation does not belong to $\Pi(V_B - c)$, we know by Properties 4.1 and 4.3 that the optimal allocation we are looking for is at the junction of $m^{PC}(t; V_B - c)$ and $m^{TR}(t)$. The optimal tax schedule is thus the solution in (t, m) of the system of equations (4.33)–(4.34).

When income taxes can be non-linear, the government's programme of optimization consists in determining z_A and x_A which maximize its social objective subject to

$$\begin{cases} V_A'(\theta) = -\dfrac{z_A(\theta)}{\theta} u_z'(x_A(\theta), z_A(\theta); \theta) \\ z_A'(\theta) \geq 0 \\ V_A(\theta) \geq V_B(\theta) - c(\theta) \end{cases} \quad \text{for all } 0 \leq \theta \leq \widehat{\theta}, \tag{4.40}$$

and to the tax revenue constraint (TR). The constraints in (4.40) are the first and second-order conditions for incentive compatibility and the participation constraints respectively. We employ a set of sufficient conditions in order to build the optimal tax schemes. The main difficulties are that (i) the participation constraints can be binding in any subset of $\left[\theta, \widehat{\theta}\right]$, even at isolated points; (ii) the adjoint variable associated with the first-order condition for incentive compatibility can have jump discontinuities. In fact, it turns out that when migration costs are constant, we manage to construct optimal tax schemes which have the following property.

Assumption 4.6 *Under the optimal tax scheme, there is no bunching ($z_A' > 0$), the adjoint variable associated with the first-order condition for incentive compatibility is continuous, and there is a productivity level θ_* from which the participation constraints are binding.*

Lemma 1 in Simula and Trannoy (2006c) states the sufficient conditions for a tax scheme to be optimal provided 4.6 holds. Manipulating these conditions, it is possible to establish that the optimal marginal tax rates are given by

$$\frac{T'}{1 - T'} = \left(1 + \frac{1}{e^H(\theta)}\right) \frac{F(\theta_*) - F(\theta)}{\theta f(\theta)} \text{ for } \theta < \theta_* \text{ and } 0 \text{ for } \theta_* \leq \theta \leq \widehat{\theta}, \tag{4.41}$$

when migration costs are constant. We then proceed as follows to compute the optimal allocation. (1.) We choose a value of θ_*. (2.) We use (4.41) to compute T', and derive l_A. (3.) We integrate the first-order condition for incentive-compatibility between 0 and θ to obtain

$$V_A(\theta) = V_A(0) + \int_0^\theta V_A'(\tau)\, d\tau. \tag{4.42}$$

We note that, by (4.30), $T(\theta l_A) = \theta l_A - V_A - v(l_A)$ and substitute this expression in the the tax revenue constraint to get $V_A(0)$. Plugging $V_A(0)$ into (4.42), we obtain $V_A(\theta)$ for all $\theta \leq \widehat{\theta}$. (4.) We check that the participation constraints are satisfied. If it is not the case, we go back to (1.) and change the value of θ_* by an increment. (5.) We check that the other sufficient conditions and Assumption 4.6 are satisfied. If such is the case, our candidate tax scheme is socially optimal given $\widehat{\theta}$. Further details are given in Simula and Trannoy (2006c).

To determine the optimum $\widehat{\theta}$ under the Resident criterion, we resort to the methodology described above to compute social welfare for a large number of $\widehat{\theta} \in \left[0, \overline{\theta}\right]$.

CHAPITRE 5

L'IMPÔT NON-LINÉAIRE OPTIMAL LORSQUE LES AGENTS TALENTUEUX VOTENT AVEC LEURS PIEDS

5.1. INTRODUCTION[1]

In Mirrlees's (1971) seminal article, migration is supposed to be impossible. However, according to Mirrlees himself, "since the threat of migration is a major influence on the degree of progression in actual tax systems, at any rate outside the United States, this is [an] assumption one would rather not make". This threat is certainly even more topical after 36 years of increasing international openness. A first source of worry in many OECD countries is the departure of some of their highly skilled individuals for tax havens (OECD, 2002). A second source of concern is the challenge faced by some developed high-tax countries because their neighbours have less redistributive objectives. For instance, about 34 000 income taxpayers have left France each year since 2000. These individuals paid three times more taxes than the average taxpayer and 70% of them chose to relocate to another EU country, like the UK, Belgium, Luxembourg, Switzerland, or to North America (DGI, 2005), mainly to countries where income taxes are lower. Half of the French living abroad earns more than 45 000 euros per year. According to the German chamber of commerce, the same story applies to Germany which was left by 145 000 income taxpayers

[1] This chapter is joint work with Alain Trannoy. We are particularly indebted to Tony Atkinson, Guy Laroque and John Weymark for very detailed and insightful comments. We are grateful to Thomas Aronsson, David Bevan, Chuck Blackorby, Philippe Choné, David de la Croix, Jeremy Edwards, Jonathan Hamilton, Jean Hindriks, Mathias Hungerbühler, Laurence Jacquet, Karolina Kaiser, Jean-Marie Lozachmeur, Hamish Low, François Maniquet, Michael S. Michael, James Mirrlees, Gareth Myles, Frank Page, Pierre Pestieau, Thomas Piketty, Panu Poutvaara, Emanuela Sciubba, and David Wildasin for their suggestions. Thanks are also due to Hélène Couprie and Gwenola Trotin. Our work has benefited from comments of seminar participants at GREQAM, the University of Cambridge, PET, ESWC, ASSET, HECER Workshop on Fiscal Federalism, Séminaire Economique de Louvain, Doctorales ADRES, Journées Louis-André Gérard-Varet, ESEM, IIPF Congress, EDGE Jamboree. The usual caveats apply.

in 2005. These examples suggest that international differences in income taxes are *one* of the determinants of the migration decision. This motivation for leaving the home country is in accordance with John Hicks's idea that migration decisions are based on the comparison of earnings opportunities across countries, net of moving costs, which is the cornerstone of practically all modern economic studies of migration (Sjaastad, 1962; Borjas, 1999).

The mobility of highly skilled individuals for tax purposes induces both losses in taxes and in productive capacity in the left countries. It differs from the brain drain because its key parameter is not the change in productivity resulting from emigration. Governments have also a more limited set of instruments than when they face tax evasion (e.g. Sandmo (1981); Chander and Wilde (1998); Slemrod and Kopczuk (2002)). They have indeed few alternatives but to reduce taxes to prevent the departure of highly skilled individuals : in a nutshell, they can use "carrots" but no "sticks". As a result, a specific conflict arises between the desire to maintain national income per capita in keeping taxes down and the aim to sustain the redistribution programme. The possibility that highly skilled individuals vote with their feet with a view to paying lower taxes appears therefore as a *new constraint* on the design of the optimal income tax schemes.

This paper studies the optimal non-linear income tax in a Mirrleesian economy ("home country") the citizens of which have type-dependent outside options consisting in emigrating to a *less* redistributive country ("foreign country"). The government wants to redistribute incomes from the more to the less productive individuals as in Mirrlees's (1971) model, but has also to take account of participation constraints for the individuals it wants to keep at home. The optimal income tax papers taking individual mobility into account have used models with no leisure-consumption choice (Mirrlees, 1982; Hindriks, 1999; Osmundsen, 1999), considered a world with two classes of individuals and lump-sum taxes (Leite-Monteiro, 1997), focused on linear taxes (Wilson, 1980, 1982c; Simula and Trannoy, 2006b), or employed Stiglitz's (1982) self-selection approach with two types of individuals (Huber, 1999; Hamilton and Pestieau, 2005; Piaser, 2003). Among them, Leite-Monteiro (1997), Hindriks (1999), Huber (1999) and Piaser (2003) have adopted the point of view of tax competition. Hamilton and Pestieau (2005) have concentrated on migration equilibria.

This paper considers optimal non-linear income taxation when there is a continuum of individuals differing in productivity as well as migration costs and facing consumption-leisure choices in the absence of unemployment. It examines how the foreign income tax policy influences the optimal income tax schedule implemented at home when agents vote with their feet. Since at this stage we do not want to study the reverse question, the foreign tax policy has to be exogenous with respect to the policy implemented at home[2]. The only coherent case is that in which the foreign government chooses the laissez-faire. Otherwise, the tax revenue constraint abroad would usually be slack or violated after the arrival of individuals from the home country, so the income tax scheme abroad should be adapted[3]. This does not seem to egregious since the focus

[2]Studying the interaction between two non-linear income tax countries raises difficult problems. For instance the revelation principle would generally vanish in A and B (See Page and Monteiro (2003) for non-linear pricing).

[3]It should be noted that the main properties derived below remain valid when B's government implements a

is on highly skilled emigration caused by a significant asymmetry in tax levels between home and abroad. In addition, it is considered that foreigners do not emigrate to the home country, for instance because of large moving costs or legal barriers. Finally, we assume that both countries have the same constant-returns-to-scale production function as we do not want individual productivity to depend on the country of residence.

The *social objective* is more complex to specify when individuals are allowed to vote with their feet because the set of agents whose welfare is to count can depend on the income tax itself. We distinguish three social criteria. Under the National criterion, the domestic government maximizes the average welfare of its citizens whilst ensuring that every citizen lives at home. Under the Citizen criterion, it maximizes the average welfare of its citizens, irrespective of their country of residence. Under the Resident criterion, it maximizes the average welfare of its residents.

We consider that an individual chooses to emigrate if his indirect utility at home is lower than his best outside option. Since many empirical studies have shown that the propensity to migrate increases with the skill level (Sahota, 1968; Schwartz, 1973; Gordon and McCormick, 1981; Nakosteen and Zimmer, 1980; Inoki and Surugan, 1981; Hanson, 2005), it is sensible to assume that more productive individuals should have more attractive outside options. Consequently, the reservation utility, i.e. the minimum utility the domestic government should give to keep an individual at home, should be increasing in productivity. We ensure it is the case by assuming that the costs of migration, expressed in terms of utility, depend on productivity and do not increase faster than the indirect utility in B. In addition, it is considered that the government knows the distribution of these costs, which implies that productivity is the sole parameter of heterogeneity within the population.

Since individuals have type-dependent outside options, A's optimal income tax scheme must satisfy *type-dependent participation constraints*. We borrow these constraints from contract theory (see Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995), and Jullien (2000)) and introduce them in the optimal non-linear income tax problem à la Mirrlees to examine at which productivity levels they should be binding to obtain the highest social welfare from our three social criteria[4]. We have shown in Simula and Trannoy (2006b) that linear taxes lack degrees of freedom when such constraints are taken into account, which explains why we consider non-linear taxes herein.

Our *main findings* can be summarized as follows. When each individual's productivity is public information (*first-best*), it is socially optimal to prevent emigration of the highly-skilled individuals under the Citizen criterion, which coincides therefore with the National criterion at the optimum. By contrast, emigration of highly-skilled workers can be socially optimal under the Resident criterion. In every case, there is a curse of the middle-skilled workers at the optimum,

given non-linear income tax schedule. What really matters is that B's tax policy is given. We only assume that B is a laissez-faire country for theoretical coherence.

[4] See Osmundsen (1999) for a direct application of the framework developed by Maggi and Rodriguez-Clare (1995) to a tax problem in a country the individuals of which share their working time between home and abroad. By construction, this model does not allow to investigate how individual mobility alters the issues raised in the closed-economy optimal income tax literature.

instead of the curse of the highly skilled obtained in closed economy (Mirrlees, 1974). Indeed, it is no longer possible to demand as much work as without mobility from the highly skilled individuals, so the productive rent is extracted to the maximum from the most productive individuals among those insufficiently talented to threaten to emigrate. However, these middle-skilled workers cannot be taxed at will because they would otherwise threaten to emigrate. Consequently, the redistribution in favour of the low-skilled individuals has to be reduced.

When each individual's productivity is private information (*second-best*), two qualitative properties of the optimal marginal tax rates are lost : they can be non-positive at interior points and strictly negative at the top. Consequently, individual mobility does not only render the tax schedule less progressive, but can also make the tax function *decreasing*. In fact, the small tax reform perturbation around the optimal tax scheme used by Piketty (1997) and Saez (2001) has an additional *participation effect* on social welfare, which favours a decrease in the optimal marginal tax rates even for individuals below the productivity levels where the individuals threaten to emigrate. This new effect results in changes in Mirrlees's (1971) and Diamond's (1998) formulae to ensure that the optimal *average* tax rates are compatible with the participation constraints of the individuals threatening to emigrate. In addition, the interaction between the type-dependent participation constraints and the incentive compatibility conditions can give rise to *countervailing incentives*, in which case less skilled individuals want to mimic more skilled individuals because the latter have more appealing outside options. Countervailing incentives cause an indirect social cost of the presence in A of the highly-skilled individuals. The Citizen and Resident criteria allow us to consider whether it is not too expensive in terms of social welfare to implement a tax scheme which prevents emigration of the highly skilled workers. When the indirect cost due to countervailing incentives prevails over the benefits of them staying in A, implementing a tax schedule inducing them to emigrate increases social welfare.

Numerical simulations calibrated with French data are provided to *quantify* to which extent individual mobility alters the whole optimal non-linear income tax schedule. They emphasize that the optimal marginal and average tax rates are significantly altered even if there are very few people threatening to emigrate. In particular, the optimal average tax rates can start to decrease far below the income level from which potential mobility occurs. There is consequently a *second-best curse of the middle-skilled*, consisting in them being taxed the most in proportion to gross income. This curse is even stronger when migration costs are decreasing in productivity : the optimal income tax schedule is not only less progressive but also such that the highly-skilled pay taxes lower than the middle-skilled.

The paper is organized as follows. Section 2 sets up the model. Section 3 examines the first-best optimal allocations. Section 4 studies the properties of the second-best optimal allocations. Section 5 provides numerical simulations on French data. Section 6 concludes. Most proofs are relegated to the Appendix.

5.2. THE MODEL

The world consists of two countries, A and B. All individuals are initially living in A. A's government implements a redistributive tax policy and B is committed to being a laissez-faire country. The governments provide no public goods. Both countries have the same production function with constant returns to scale. Hence, productivity levels, equal to wages in the absence of taxation, are independent of the country in which the individuals are working.

Individuals differ in productivity θ. An individual with productivity θ is called a θ-individual. The cumulative distribution function of θ, denoted F, is common knowledge. It is defined on a closed interval $\left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}^+$ where it admits a continuous and strictly positive density f.

5.2.1. Individual Behaviour

All individuals have the same preferences over consumption x and labour ℓ. If $\overline{\ell}$ is the time endowment, these preferences are represented by a cardinal utility function $U : \mathcal{X} \to \mathbb{R}$, where $\mathcal{X} := \left\{(x, \ell) \in \mathbb{R}^+ \times [0, \overline{\ell})\right\}$.

Assumption 5.1 *U is a $\mathcal{C}^2$ strictly concave function such that $U_x > 0$, $U_\ell < 0$ and $U \to -\infty$ as $x \overset{>}{\to} 0$ or $\ell \overset{<}{\to} \overline{\ell}$.*

Assumption 5.2 *Leisure is a normal good.*

A θ-individual working ℓ units of time has gross income $z := \theta\ell$. We call

$$u(x, z; \theta) := U(x, z/\theta) \tag{5.1}$$

the personalized utility function and note that $u'_x = U'_x$, $u'_z = U'_\ell/\theta$, $u''_{xx} = U''_{xx}$, $u''_{xz} = U''_{x\ell}/\theta$, $u''_{zz} = U''_{\ell\ell}/\theta^2$. The marginal rate of substitution of gross income for consumption of a θ-individual at (x, z) is

$$s(x, z; \theta) := -\frac{u'_z(x, z; \theta)}{u'_x(x, z; \theta)}. \tag{5.2}$$

Each individual decides about the optimal amount of consumption and labour to maximize his utility subject to his budget constraint. The government uses a tax function $T(\theta, \ell)$, with $T(\theta, \ell) = T(\theta)$ in the first-best and $T(\theta, \ell) = T(\theta\ell)$ in the second-best. Using T, A's government can arrange that an individual with gross income z has disposable income $z - T(\theta, \ell)$ in A. Consequently, the utility maximization programme in A, $\max_{(x,\ell)\in\mathcal{X}} \{U(x, \ell) \text{ s.t. } x = z - T(\theta, \ell)\}$, defines implicitly the consumption and labour supply functions in A, $x_A(\theta)$ and $\ell_A(\theta)$ respectively. The indirect utility in A is

$$V_A(\theta) := U(x_A(\theta), \ell_A(\theta)). \tag{5.3}$$

We call e^H and e^M the Hicksian and Marshallian elasticities of labour supply with respect to the net-of-tax wage rate.

The utility maximization programme in B, $\max_{(x,\ell)\in\mathcal{X}}\{U(x,\ell) \text{ s.t. } x=z\}$, defines implicitly the consumption and labour supply functions in B, $x_B(\theta)$ and $\ell_B(\theta)$ respectively. The indirect utility in B is

$$V_B(\theta) := U(x_B(\theta), \ell_B(\theta)), \tag{5.4}$$

which is strictly increasing in θ.

5.2.2. Emigration and Participation Constraints

An individual leaving A pays a strictly positive *migration cost* c. Given the cardinality of individual preferences, this cost can be expressed as a "time-equivalent" loss in utility, due to different material and psychic costs of moving : application fees, transportation of persons and household's goods, forgone earnings, costs of speaking a different language and adapting to another culture, costs of leaving one's family and friends, etc. "[These migration] costs probably vary among persons [but] the sign of the correlation between costs and wages is ambiguous" (Borjas, 1999, p. 12). We consider that they depend on productivity and that their distribution is known to A's government. Hence, A's government knows $c(\theta)$ when it knows θ, which is thus the sole parameter of heterogeneity within the population. In addition :

Assumption 5.3 $c:\left[\underline{\theta},\overline{\theta}\right]\to\mathbb{R}^{++}$ *is a* $\mathcal{C}^2$ *function satisfying* $c'(\theta) < V_B'(\theta)$.

The *reservation utility* is the maximum utility an individual staying in A can obtain abroad. It is thus equal to $V_B(\theta) - c(\theta)$. Assumption 5.3 amounts therefore to considering that the outside opportunities are increasing in productivity. This is in accordance with many empirical studies, which find that the propensity to migrate increases with productivity (Sahota, 1968; Schwartz, 1973; Gordon and McCormick, 1981; Nakosteen and Zimmer, 1980; Inoki and Surugan, 1981; Hanson, 2005; Docquier and Marfouk, 2005). For instance, within the EU, the migration rate of the skilled population is 8.1% versus 4.8 for the unskilled one (Docquier and Marfouk, 2005). It thus does not seem too egregious to consider that more skilled individuals should have more attractive outside options, i.e. higher reservation utilities. Moreover, Assumption 5.3 places *no* restriction on the *level* of the migration costs, expect that they are positive.

The *location rent* of a θ-individual is the excess of his indirect utility in A over his reservation utility, i.e.

$$R(\theta) = V_A(\theta) - V_B(\theta) + c(\theta). \tag{5.5}$$

An individual stays in A if and only if

$$R(\theta) \geq 0, \tag{5.6}$$

and therefore leaves A if and only if $R(\theta) < 0$.

A *citizen* is defined as an individual born in A, so all individuals have A's citizenship. Individuals are committed to working in the country where they live. Since the focus is on emigration of highly skilled individuals, we consider that there is a partition of citizens between A and B, with the low-skilled individuals being in A. In addition, emigration of a set of individuals of measure zero does not capture any economic phenomenon, so we assume that A's resident population is compact.

Assumption 5.4 *A's resident population is a closed interval of types* $\left[\underline{\theta}, \widehat{\theta}\right]$, *with* $\widehat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$.[5]

We consider that A's government is not able to levy taxes in B, since the fiscal prerogative is closely linked to national sovereignty, and not willing to redistribute income to the individuals staying in B. Consequently, $T : \mathcal{T} \rightarrow \mathbb{R}$ with $\mathcal{T} = \left[\underline{\theta}, \widehat{\theta}\right] \times \left[0, \overline{\ell}\right]$. Since $T := z_A - x_A$, a tax policy is *budget balanced* if and only if it satisfies the tax revenue constraint

$$\int_{\underline{\theta}}^{\widehat{\theta}} (z_A - x_A)\, dF(\theta) \geq 0. \tag{TR}$$

In the rest of the paper, we denote by γ the Lagrange multiplier associated with (TR).

5.2.3. Social Criteria

A's government is a benevolent policy maker which intends to implement the tax policy corresponding to the best compromise between equity and efficiency. Its desire to redistribute income is captured through its aversion to income inequality $\rho \in \mathbb{R}^{+}$. A zero aversion corresponds to utilitarianism and an infinite one to the Rawlsian maximin.

The social objective is more difficult to specify than in closed economy. Indeed, it does not only depend on ρ which is captured through an isoelastic function defined by $\phi_\rho : \mathbb{R}^{++} \rightarrow \mathbb{R}$, $\phi_\rho(U) = U^{1-\rho}/(1-\rho)$ for $\rho \neq 1$ and $\phi_1(U) = \ln U$ for $\rho = 1$, but also on the answers to the following questions. First, should we maximize total or average social welfare ? Classical utilitarianism has been criticized on the ground that it leads to the so-called repugnant conclusion and this is a significant shortcoming (For details, see Blackorby, Bossert, and Donaldson (2005)). Average utilitarianism does not suffer from this drawback. So, we consider that the government is interested in social welfare per capita, which allows us to compare allocations differing in population size. Second, who are the agents whose welfare is to count ? At least three social criteria can be proposed, each of which corresponds to a specific answer.

Under the *National criterion*, A'*s* government cares about the welfare of all its citizens and wants each citizen to choose to stay in A. The social objective is

$$W^N_{A,\rho} := \int_{\underline{\theta}}^{\widehat{\theta}} \phi_\rho(V_A(\theta))\, dF(\theta), \text{ with } W^N_{A,\rho} = -\infty \text{ for } \widehat{\theta} < \overline{\theta}. \tag{5.7}$$

This objective corresponds to the mercantilist idea, formulated by Bodin (1578), that "the only source of welfare is mankind itself". Emigration should therefore be prevented to keep the state

[5] This assumption is a restriction on the class of feasible tax schedules. In the absence of this assumption, the resident population in A might consist of several disjoint intervals. It thus not seem possible to address such issues with optimal control theory (Seierstad and Sydsaeter, 1987).

prosperous. This social criterion is considered to provide a building block for the solutions of the following more appealing Citizen and Resident criteria.

Under the *Citizen criterion*, A's government cares about the average social welfare of its citizens, whether they are in A or B. Under Assumption 5.4, the social objective is

$$W_{A,\rho}^{C}\left(\widehat{\theta}\right):=\int_{\underline{\theta}}^{\widehat{\theta}}\phi_{\rho}\left(V_{A}\left(\theta\right)\right)dF\left(\theta\right)+\int_{\widehat{\theta}}^{\overline{\theta}}\phi_{\rho}\left(V_{B}\left(\theta\right)-c\left(\theta\right)\right)dF\left(\theta\right). \tag{5.8}$$

This criterion rests on the idea that the fiscal system finds its legitimacy in its democratic adoption. Consequently, the welfare of every individual who has the right to vote should be taken into account, irrespective of his country of residence[6]. When this objective is chosen, the optimal tax function depends on the choice of $\widehat{\theta}$ and determines an allocation of A's citizens between A and B. Hence, A's resident population is endogenous while the set of agents the welfare of whom matters is exogenously fixed.

Under the *Resident criterion*, A's government cares about the average social welfare of its residents. Under Assumption 5.4, the social objective is

$$W_{A,\rho}^{R}\left(\widehat{\theta}\right):=\frac{1}{F\left(\widehat{\theta}\right)}\int_{\underline{\theta}}^{\widehat{\theta}}\phi_{\rho}\left(V_{A}\left(\theta\right)\right)dF\left(\theta\right). \tag{5.9}$$

This criterion is based on the idea that a public policy should take the welfare of all taxpayers into account. Consequently, the welfare of the citizens living in B does not count. When this objective is chosen, the tax function as well as the set of agents whose welfare is to count depend on the choice of $\widehat{\theta}$.[7] $W_{A,\rho}^{R}\left(\widehat{\theta}\right)$ is based on average utilitarianism, which is known to face the Mere Addition Paradox : the addition of individuals whose utility is less than the average utility in the initial population is regarded as suboptimal even if this change in population size affects no one else and does not involve social injustice. In the second-best framework, this paradox does not really matter herein because we are focusing on emigration of the highest skilled individuals initially living in A, whose utility is greater than the maximum utility in A.

5.3. FIRST-BEST OPTIMAL ALLOCATIONS

This section characterizes the first-best optimal allocations where each individual's productivity is public information. Consequently, A's government implements a tax policy depending on productivity, i.e. $T(\theta,\ell)=T(\theta)$. We restrict attention to the tax schedules which are continuous and differentiable almost everywhere.

[6] In France, the 14th Article of the Declaration of the Rights of Man and of the Citizen, which has constitutional value, provides that : "*All citizens* have the right to vote, by themselves or through their representatives, for the need for the public contribution, to agree to it voluntarily, to allow implementation of it, and to determine its appropriation, the amount of assessment, its collection and its duration". For example, twelve senators represent the French citizens living abroad.

[7] In other words, a population problem consisting in "*different* number choices" (Parfit, 1984) is embedded in the optimal income tax problem.

The indirect utility if A were a closed economy, $V_A^{c\ell}(\theta)$, is used as a benchmark. When ρ is finite, it is decreasing in θ at the social optimum if and only if Assumption 5.2 holds (Mirrlees, 1974) : there is therefore a *curse of the highly skilled workers*. When ρ is infinite, all individuals receive the same utility level. In this section, we assume $V_A^{c\ell}\left(\overline{\theta}\right) < V_B\left(\overline{\theta}\right) - c\left(\overline{\theta}\right)$ since otherwise the participation constraints would never be active. It is worth noting that, in the first-best setting, individuals for whom the participation constraints are active pay strictly positive taxes. Indeed, since $V_B(\theta) - c(\theta) < V_B(\theta)$ under Assumption 5.3, their budget constraints must be below the 45°-line through the origin in the gross-income/consumption space.

5.3.1. National criterion

A's government chooses the tax paid by each individual or, equivalently, the consumption-labour bundle intended for each individual.

Problem 5.1 (National Criterion, First-Best) *Find* $(x, \ell) \in \mathcal{X}$ *to maximize* $W_{A,\rho}^N$, *with* $\widehat{\theta} = \overline{\theta}$, *subject to (TR) and*

$$R(\theta) \geq 0 \text{ for } \theta \leq \widehat{\theta}. \tag{PC}$$

When the set $\left\{\theta \in \left[\underline{\theta}, \overline{\theta}\right] : (5.6) \text{ binding}\right\}$ is non-empty, we call θ^* its *infimum*.

Proposition 5.1 (The Curse of the Middle-Skilled) *The participation constraints are binding from* $\theta^* < \overline{\theta}$. *When* $\rho < \infty$, *the optimum indirect utility in A is V-shaped in* θ, *minimum at* θ^*. *When* $\rho \to \infty$, *the optimum indirect utility in A is constant up to* θ^* *and then increasing.*

Proof. See 5.7.1 in the Appendix. ■

Figure 5.1 illustrates Proposition 5.1. On panel (a), the government's aversion to income inequality is finite. The θ^*-individuals are the worse-off when potential mobility is taken into account. On panel (b), the government is Rawlsian. The utility levels of the individuals with productivity below θ^* are reduced compared to the closed economy.

The participation constraints (PC) separate the population into two intervals : they are inactive below θ^* and active above. Consequently, it is no longer possible to require the most talented individuals to work as much as without mobility, i.e. to require them to keep working even though labour disutility exceeds the gains from the increase in income. The productive rent is thus extracted to the maximum from the most productive individuals among those threatening to emigrate. Redistribution in A is reduced and the situation of the low-skilled individuals gets worse.

It is therefore from the most productive individuals among those insufficiently talented to threaten to leave the country that the productive rent is extracted to the maximum. However, this rent cannot be extracted at will because of the participation constraints. Redistribution in A is thus reduced and the situation of the low-skilled individuals deteriorates.

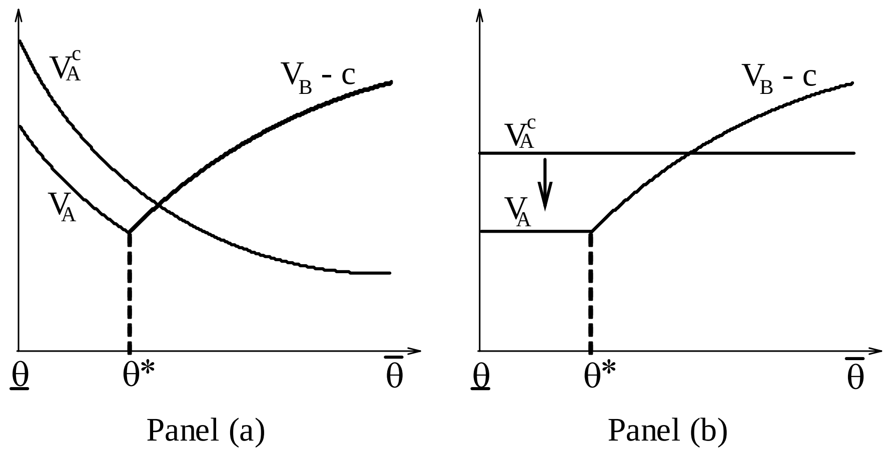

FIG. 5.1 – The curse of the middle-skilled workers

5.3.2. Citizen Criterion

We examine if it is socially optimal to prevent emigration of the highly skilled individuals under the Citizen criterion.

Problem 5.2 (Citizen Criterion, First-Best) *Find* $(x, \ell) \in \mathcal{X}$ *and* $\widehat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$ *to maximize* $W^C_{A,\rho}\left(\widehat{\theta}\right)$ *subject to (PC) and (TR).*

Proposition 5.2 *Under the Citizen criterion, the optimal tax policy is the same as that chosen under the National criterion.*

The proof proceeds by contradiction. Assume $\widehat{\theta} < \overline{\theta}$ is socially optimal. The individuals with productivity $\widehat{\theta}$ are indifferent between A and B, i.e. $R\left(\widehat{\theta}\right) = 0$, and those with productivity greater than $\widehat{\theta}$ emigrate to B. It is always *feasible* to make the latter relocate to A, without reducing the indirect utilities of A's residents, in giving them their laissez-faire utility V_B (or a bit more than their reservation utility). Since $c\left(.\right) > 0$ and $\phi'_\rho\left(.\right) > 0$, one gets $\phi_\rho\left(V_B\left(.\right) - c\left(.\right)\right) < \phi_\rho\left(V_B\left(.\right)\right)$ and thus

$$\int_{\underline{\theta}}^{\widehat{\theta}} \phi_\rho\left(V_A\left(\tau\right)\right) dF\left(\tau\right) + \int_{\widehat{\theta}}^{\overline{\theta}} \phi_\rho\left(V_B\left(\tau\right)\right) dF\left(\tau\right) > \int_{\underline{\theta}}^{\widehat{\theta}} \phi_\rho\left(V_A\left(\tau\right)\right) dF\left(\tau\right) + \int_{\widehat{\theta}}^{\overline{\theta}} \phi_\rho\left(V_B\left(\tau\right) - c\left(\tau\right)\right) dF\left(\tau\right), \quad (5.10)$$

the RHS of which is $W^C_{A,\rho}\left(\widehat{\theta}\right)$. Hence, making the highly-skilled emigrate from B to A results in a feasible increase in social welfare, which contradicts the premise. The social optimum corresponds therefore to the corner solution as regards the allocation of individuals between A and B.

5.3.3. Resident Criterion

The basic difference between the citizen criterion and the resident one is that the latter does not take the welfare of A's citizens living in B. Hence, it might be socially desirable to let some individuals emigrate to B.

Problem 5.3 (Resident Criterion, First-Best) *Find* $(x, \ell) \in \mathcal{X}$ *and* $\widehat{\theta} \in [\underline{\theta}, \overline{\theta}]$ *to maximize* $W^R_{A,\rho}\left(\widehat{\theta}\right)$ *subject to (PC) and (TR).*

By the envelope theorem and Leibnitz's rule, the impact of a small increase in $\widehat{\theta}$ on the social objective is given by

$$\frac{\partial W^R_{A,\rho}\left(\widehat{\theta}\right)}{\partial \widehat{\theta}} = \gamma \frac{T\left(\widehat{\theta}\right) f\left(\widehat{\theta}\right)}{F\left(\widehat{\theta}\right)} + \frac{\left[\phi_\rho\left(V_A\left(\widehat{\theta}\right)\right) - W^R_{A,\rho}\left(\widehat{\theta}\right)\right] f\left(\widehat{\theta}\right)}{F\left(\widehat{\theta}\right)}. \tag{5.11}$$

The first term, which is strictly positive, corresponds to the fiscal contribution of the $\widehat{\theta}$-individuals $T\left(\widehat{\theta}\right) f\left(\widehat{\theta}\right)$, converted in social welfare per capita. The second term (5.11) basically compare the social utility of the marginal individuals, whose number is represented by $f\left(\widehat{\theta}\right)$, to the average social welfare. It is divided by the size of the population in order to obtain a per capita measurement. Its sign is ambiguous. As a consequence, a trade-off appears between the the "tax effect" and the "utility effect" of the presence in A of the marginal $\widehat{\theta}$-individuals. In the optimum, (5.11) must be non-negative. As a consequence, a sufficient condition for emigration of the most productive citizens to be socially optimal is obtained.

Proposition 5.3 *Under the Resident criterion, letting the most productive citizens emigrate increases social welfare when* $\partial \overline{W}^R_{A,\rho}\left(\widehat{\theta}\right) / \partial \widehat{\theta}\Big|_{\widehat{\theta}=\overline{\theta}} < 0$.

In order to determine the optimal upper bound of the resident population, $\widehat{\theta}$, the analogue of Problem 5.3 in which $\widehat{\theta}$ is arbitrarily given in $[\underline{\theta}, \overline{\theta}]$ is first considered. Let $\overline{W}^R_{A,\rho}\left(\widehat{\theta}\right)$ be the social value function. The optimal value of $\widehat{\theta}$ is that for which $\overline{W}^R_{A,\rho}\left(\widehat{\theta}\right)$ is maximum.

5.4. SECOND-BEST OPTIMAL ALLOCATIONS

The distribution of characteristics in the economy remains common knowledge, but individual productivity is now private information. A's government is thus restricted to setting taxes as a function of earnings, i.e. $T(\theta, \ell) = T(z)$. Hence, it has to ensure that the tax schedule is incentive compatible.

5.4.1. Statement of the Problem

T is an *incentive compatible* tax schedule if and only if individuals living in A have an incentive to reveal their type truthfully when it is implemented. By the revelation principle, the incentive compatibility conditions read

$$u\left(x_A\left(\theta'\right), z_A\left(\theta'\right); \theta\right) \leq u\left(x_A\left(\theta\right), z_A\left(\theta\right); \theta\right) \text{ for all } \left(\theta, \theta'\right) \in \left[\underline{\theta}, \widehat{\theta}\right]^2. \tag{IC}$$

To deal with this uncountable infinity of constraints, the Spence-Mirrlees property is assumed to hold :

Assumption 5.5 (Single-Crossing) $s'_\theta\left(x, z; \theta\right) < 0$.

Under Assumption 5.5, (IC) is equivalent to :

$$V'_A\left(\theta\right) = -\frac{z_A\left(\theta\right)}{\theta} u'_z\left(x_A\left(\theta\right), z_A\left(\theta\right); \theta\right) \text{ for } \theta \leq \widehat{\theta}, \tag{FOIC}$$

$$z_A\left(\theta\right) \text{ non-decreasing for } \theta \leq \widehat{\theta}. \tag{SOIC}$$

The proof of this equivalence is standard and is omitted. (FOIC) is an envelope condition specifying how the indirect utility V_A must locally change. Since $V'_A \geq 0$, V_A cannot be V-shaped as in the first-best. (SOIC) is a global monotonicity condition of gross income. The analysis will herein focus on continuous mechanisms which possibly exhibit kinks at a finite number of points corresponding to jumps of the marginal tax rate. In this case, $R\left(\theta\right)$ is continuous and (SOIC) is equivalent to

$$z'_A\left(\theta\right) \geq 0 \text{ for } \theta \leq \widehat{\theta}. \tag{SOIC'}$$

Since A's government does not know who are the agents for whom the location rent $R\left(\theta\right)$ is zero, the participation constraints and the incentive compatibility conditions have to be taken simultaneously into account for all A's residents[8]. The second-best optimal non-linear income tax problems read thus as follows.

Problem 5.4 (Second-Best) *Find $T\left(z_A\right)$ to maximize $W^i_{A,\rho}$, $i = \{N, C, R\}$, subject to (i) (FOIC), (SOIC'), (PC), (TR); (ii) $\widehat{\theta} = \overline{\theta}$ when $i = N$ and $\widehat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$ otherwise.*

In the closed economy version of Problem 5.4, $\widehat{\theta}$ is equal to $\overline{\theta}$ and the tax revenue constraints (TR) are not taken into account. Let $V^{c\ell}_A\left(\theta\right)$ be the optimum indirect utility. If $V^{c\ell}_A\left(\theta\right) \geq V_B\left(\theta\right) - c\left(\theta\right)$ for every $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, allowing individuals to vote with their feet does not alter the social optimum. Therefore, we herein place ourselves in the case where there are individuals for whom $V^{c\ell}_A\left(\theta\right) < V_B\left(\theta\right) - c\left(\theta\right)$ since otherwise the participation constraints would never be active.

Problem 5.4 raises three main difficulties compared to its closed-economy analogue. First, (PC) can a priori bind on any subset of the resident population, even at isolated points, because $R\left(\theta\right)$ is not necessarily monotonic. Second, (PC) are pure state constraints in the optimization problems. The adjoint variables may thus have jump discontinuities. Third, under the Citizen and Resident criteria, $\widehat{\theta}$ is free to vary between $\underline{\theta}$ and $\overline{\theta}$.

[8]If the participation constraints (PC) were not type-dependent, it would be necessary and sufficient to check that they are satisfied at $\underline{\theta}$ since (FOIC) ensures that the optimal utility path is non-decreasing.

In solving Problem 5.4, we assume that the adjoint variables have a finite number of jump discontinuities and are $\mathcal{C}^1$ elsewhere. For later reference, we call ι the adjoint variable associated with (FOIC) and $\pi' \geq 0$ the Lagrange multiplier of (PC), which corresponds to the shadow price of a marginal increase in the reservation utility at θ. Let also π, with derivative π' almost everywhere, be the non-decreasing function

$$\pi(\theta) := \pi\left(\overline{\theta}\right) - \int_{\theta}^{\overline{\theta}} \pi'(\tau)\, d\tau, \tag{5.12}$$

which gives the shadow price of a uniform marginal increase in the reservation utility for all $\theta' \geq \theta$.

5.4.2. Optimal Tax Schedule for the Individuals Threatening to Emigrate

Before looking at a specific social criterion, we derive properties which are satisfied by all optimal tax schemes for the individuals threatening to emigrate.

For this purpose, let I be an interval of positive length where (5.6) is active. By definition, for $\theta \in I$, $R(\theta) \equiv 0$ and thus $V_A'(\theta) = V_B'(\theta) - c'(\theta)$. Hence the rate of increase of the indirect utility the government has to give to the individuals so that they reveal their private information, is equal to the slope of the reservation utility on I. In addition, employing (FOIC) and rearranging yield

$$z_A(\theta) = -\theta \frac{V_B'(\theta) - c'(\theta)}{u_z'(x_A(\theta), z_A(\theta); \theta)} \text{ for } \theta \in I, \tag{5.13}$$

and by differentiation,

$$z_A'(\theta) = \frac{\left[V_B'(\theta) - c'(\theta)\right]\left\{\theta\left(u_{xz}'' x_A' + u_{\theta z}''\right) - \left(1 + \theta \frac{V_B''(\theta) - c''(\theta)}{V_B'(\theta) - c'(\theta)}\right) u_z'\right\}}{\left(u_z'\right)^2 - \theta\left(V_B'(\theta) - c'(\theta)\right) u_{zz}''} \text{ for } \theta \in I. \tag{5.14}$$

The second-order condition for incentive compatibility (SOIC') can only be satisfied on I if the curly bracket in (5.14) is non-negative. When preferences are separable ($u_{xz}'' = 0$), one gets

$$z_A'(\theta) \geq 0 \Leftrightarrow \frac{\theta u_{\theta z}''}{u_z'} \leq \left(1 + \theta \frac{V_B''(\theta) - c''(\theta)}{V_B'(\theta) - c'(\theta)}\right), \tag{5.15}$$

the LHS of which is negative since $u_{\theta z}'' > 0$ and $u_z' < 0$.

Property 5.1 *Let preferences be separable ($u_{xz}'' = 0$) and consider an interval I of positive length where (PC) is active. Then, there is* no bunching *on I when*

$$\theta \frac{V_B''(\theta) - c''(\theta)}{V_B'(\theta) - c'(\theta)} > -1 \textit{ for } \theta \in I. \tag{5.16}$$

Condition (5.16) states that the elasticity of the marginal reservation utility to the wage rate, evaluated at θ, is greater than -1 for every $\theta \in I$. To have further insight, we now turn to quasilinear-in-consumption preferences,

$$u(x,z;\theta) = x - v(z/\theta), \text{ with } v(\ell) = \ell^{1+1/e}/(1+1/e). \tag{5.17}$$

The Hicksian elasticity of labour supply is thus constant ($e^H(\theta) = e$) and, by (5.13),

$$\ell_A(\theta) = \theta^{\frac{e}{1+e}} [\theta^e - c'(\theta)]^{\frac{e}{1+e}}. \tag{5.18}$$

Since $V_A(\theta) = \theta\ell_A(\theta) - T(\theta\ell_A(\theta)) - v(\ell_A(\theta))$ and $V_B(\theta) = \theta^{1+e}/(1+e)$, $R(\theta) = 0$ on I results in

$$T(z_A(\theta)) = \theta\ell_A(\theta) - V_B(\theta) + c(\theta) - v(\ell_A(\theta)). \tag{5.19}$$

Property 5.2 *Let preferences be quasilinear in consumption, e be the constant elasticity of labour supply and I be an interval of positive length where (PC) is active. Then,*

$$T(z_A(\theta)) = \theta(\theta^e - c'(\theta)) \left[\theta^{\frac{e}{1+e}} (\theta^e - c'(\theta))^{-\frac{1}{1+e}} - \frac{e}{1+e}\right] - \frac{\theta^{1+e}}{1+e} + c(\theta). \tag{5.20}$$

On I, the optimal tax liability depends on productivity, on the cost of migration and its slope as well as on the elasticity of labour supply. From now on, we concentrate on migration cost functions $c(\theta)$ which satisfy (5.16) ; hence there is no bunching on I. The first-order condition for individual utility maximization in A yields $\ell_A(\theta) = \theta^e [1 - T']^e$. Combining this expression of $\ell_A(\theta)$ with (5.18) and solving for T', the following property is obtained.

Property 5.3 *Let preferences be quasilinear in consumption, e be the constant elasticity of labour supply, $c(\theta)$ satisfy (5.16), and I be an interval of positive length where (PC) is active. Then,*

$$\frac{T'}{1-T'} = \theta^{\frac{e}{1+e}} [\theta^e - c'(\theta)]^{-\frac{1}{1+e}} - 1 \text{ for } \theta \in I. \tag{5.21}$$

In this case, the optimal marginal tax rate on I depends on the productivity level, on the elasticity of labour supply and on the slope of the costs of migration. Its sign is as follows.

Property 5.4 *Consider the same situation as in Property 5.3. Then,*

$$T'(z_A(\theta)) \gtreqqless 0 \Leftrightarrow c'(\theta) \gtreqqless 0 \text{ for } \theta \in I. \tag{5.22}$$

When the costs of migration are *non-increasing*, the theorem stating that the optimal tax function is strictly increasing at *all* income levels (Seade, 1982) does no longer hold. When the costs of migration are *strictly decreasing* in productivity, the optimal marginal tax rates faced by the individuals threatening to emigrate are *strictly negative*[9]. This property contrasts with two results obtained in closed economy, stating that : (i) the optimal marginal tax rates are

[9] An example of optimal income tax schedule with strictly negative marginal tax rates is provided in the simulation section.

non-negative (Mirrlees, 1971) ; (ii) the optimal marginal tax rate is zero at the top (Sadka, 1976; Seade, 1977). The next corollaries of Property 5.4 provide further details about these significant changes.

The first one considers constant migration costs on I. By Property 5.4, $\ell_A(\theta) = \theta^e$ on I. Moreover, $V_A = V_B - c = \theta^{1+e}/(1+e) - c$. Then, by (5.19), $T(z_A(\theta)) = c(\theta)$ on I.

Corollary 5.1 *Consider the same situation as in Property 5.3 and let $c'(\theta) = 0$ on I. Then, the optimal tax function has a flat section corresponding to potentially mobile individuals paying* taxes equal to their positive costs of migration.

Hence, because of the threat of migration, the optimal tax schedule becomes regressive : highly skilled individuals for whom the participation constraints are binding pay less taxes in proportion to gross income than lower skilled individuals. The situation is even more acute when the costs of migration are strictly decreasing.

Corollary 5.2 *Consider the same situation as in Property 5.3 and let $c'(\theta) < 0$ on I. Then, the optimal average tax rate and the optimal tax function are strictly decreasing in productivity on I.*[10]

Here, progressivity of the optimal tax schedule does not only collapse because of potential mobility ; the *tax liability* itself becomes strictly decreasing. This means that there are middle-skilled individuals insufficiently talented to leave the country which pay higher taxes than more productive individuals. This is a *second-best counterpart of the curse of the middle-skilled*, in which taxes replace utility levels.

5.4.3. National Criterion

We study the impact of the threat of migration on the optimum tax scheme in A when A's government adopts the National criterion. To this aim, Mirrlees's formula is extended to the case where agents are allowed to vote with their feet. This formula gives the optimal marginal tax rates in the absence of bunching.

A First Pass

We first look at a very simple situation to illuminate the basic economic relations which determine the optimal marginal tax rates[11]. A's government adopts the Rawlsian maximin and there are agents with zero productivity ($\underline{\theta} = 0$). The social objective is thus to maximize the social benefit given to the latter individuals. Preferences are quasilinear in consumption, which captures the absence of income effects on labour supply, and the elasticity of labour supply is

[10] The fact that the optimal average tax rate $T(z_A(\theta))/z_A(\theta)$ is strictly decreasing on I follows from $T'(z_A(\theta)) < 0$ and $z'_A(\theta) > 0$ on I.

[11] The simplifying assumptions are made to get the flavour of the result stated in Proposition 5.5 in the most general case. In addition, optimal tax schemes satisfying all of them are shown to exist in the simulation section.

constant ($e^H(\theta) = e$). Migration costs are constant, equal to $c(\theta) = \overline{c}$. In addition, attention is restricted to the cases where (PC) is only active on non-degenerate intervals ; by Property 5.1, taxes paid by individuals threatening to emigrate amount to $\overline{c}$.

We adopt the methodology employed by Piketty (1997) and Saez (2001) to derive the optimal marginal tax rates. We consider the effects of a small increase dT in the optimal marginal tax rates for income between z and $z + dz$. This tax perturbation has three effects on social welfare, captured through changes in tax revenue G. The first two effects are the same as in a closed economy. The third one is new.

Mechanical effect : All individuals with income greater than z pay additional taxes $dTdz$. Since their proportion is given by $1 - F(\theta_z)$, the effect on tax revenue is

$$dG^+ = (1 - F(\theta_z)) \times dTdz. \tag{5.23}$$

Elasticity effect : The net-of-tax wage rate of the individuals with income between z_A and $z_A + dz_A$ decreases from $\theta_z(1 - T')$ to $\theta_z(1 - T' - dT)$, i.e. by $dT/(1 - T')\%$. The reduction in gross income z for the $f\,d\theta$ individuals is therefore $e \times dT/(1 - T') \times z\,f\,d\theta$. This results in a loss in tax revenue $dG_1^- = T' \times e \times dT/(1 - T') \times zfd\theta$; since $d\theta = dz/[\ell(1 + e)]$ by definition of e,

$$dG_1^- = \frac{T'}{1 - T'}\frac{e}{1 + e} \times \theta f \times dTdz. \tag{5.24}$$

Participation effect : The individuals who are not threatening to emigrate have a strictly positive location rent $R(\theta)$ in A, i.e $V_A(\theta) > V_B(\theta) - c(\theta)$. By continuity, the strict inequality remains satisfied once the small tax reform has taken place since $V_A(\theta)$ is only marginally altered. In contrast, the individuals with productivity above θ_z who were already on their participation constraints have to be compensated for the increase in taxes they face because their location rent would otherwise be negative. Let $\mu_f\left(\left[\theta_z, \overline{\theta}\right] \cap \Theta^{PC}\right)$ represent the number of the latter. Since preferences are quasilinear in consumption and migration costs constant, each of them must receive $dTdz$. So, the overall effect on tax revenue is

$$dG_2^- = \mu_f\left(\left[\theta_z, \overline{\theta}\right] \cap \Theta^{PC}\right) \times dTdz. \tag{5.25}$$

At the social optimum, the small tax reform perturbation has no first-order effect. Consequently $dG^+ = dG_1^- + dG_2^-$. Since θ_z has been chosen arbitrarily, the following result is obtained.

Proposition 5.4 *Assume (PC) is not active at isolated points. Then, when preferences are quasilinear in consumption, $e^H(\theta) = e$ and $c(\theta) = \overline{c}$, the Rawlsian optimal marginal tax rates are given by*

$$\frac{T'}{1 - T'} = \left(1 + \frac{1}{e}\right)\frac{1 - F(\theta)}{\theta f(\theta)}\left(1 - \frac{\mu_f\left(\left[\theta, \overline{\theta}\right] \cap \Theta^{PC}\right)}{1 - F(\theta)}\right) \text{ for } \theta < \overline{\theta}, \tag{5.26}$$

and $T' = 0$ for $\theta = \overline{\theta}$, provided there is no bunching at the optimum.

Proof. See 5.7.2 in the Appendix for the formal proof. ■

When $\mu_f\left(\left[\theta,\overline{\theta}\right]\cap\Theta^{PC}\right) = 0$, (5.26) reduces to the formula derived by Piketty (1997) in closed economy. Three points are worth noting. First, if the participation constraints are active at any given $\theta < \overline{\theta}$, $T'\left(z_A\left(\theta\right)\right) = 0$ by Property 5.3. But, by (5.26), $T'\left(z_A\left(\theta\right)\right) = 0$ if and only if $\mu_f\left(\left[\theta,\overline{\theta}\right]\cap\Theta^{PC}\right) = 1 - F\left(\theta\right)$, that is if and only if the participation constraints are active for all individuals with productivity greater than θ. So, the participation constraints separate the population into two intervals : they are slack for $\theta < \theta^*$ and binding for $\theta \geq \theta^*$. Hence, Proposition 5.4 can alternatively be formulated as follows.

Corollary 5.3 *Consider the same situation as in Proposition 5.4. Then,*
(i) $T'/\left(1-T'\right)$ is given by (5.26) for $\theta < \theta^*$;
(ii) θ^* is the minimum productivity level for which $T'\left(z_A\left(\theta\right)\right) = 0$;
(iii) $T\left(\theta\right) = c\left(\theta\right)$ for $\theta \geq \theta^*$.

Second, $\mu_f\left(\left[\theta,\overline{\theta}\right]\cap\Theta^{PC}\right)/\left(1-F\left(\theta\right)\right)$ is increasing in θ for $\theta \leq \theta^*$. Accordingly, the closer θ to θ^*, the greater the reduction in marginal tax rates. Third, since e^H is constant, (5.26) can be rewritten as

$$\frac{T'}{1-T'} = \frac{T'_{c\ell}}{1-T'_{c\ell}}\left(1 - \frac{\mu_f\left(\left[\theta,\overline{\theta}\right]\cap\Theta^{PC}\right)}{1-F\left(\theta\right)}\right), \tag{5.27}$$

where $T'_{c\ell}$ is the Rawlsian marginal tax rate the θ-individuals would face in A in the absence of individual mobility. As $\mu_f\left(\left[\theta,\overline{\theta}\right]\cap\Theta^{PC}\right) \leq 1 - F\left(\theta\right)$, the marginal tax rates faced by *all* individuals, and not only those of the individuals threatening to emigrate are reduced in the presence of potential mobility.[12]

The General Case

We extend the previous analysis by relaxing all simplifying assumptions, except the absence of bunching.

Proposition 5.5 *Under the National criterion and in the absence of bunching, the optimal marginal tax rates are given by*

$$\frac{T'\left(z_A\left(\theta\right)\right)}{1-T'\left(z_A\left(\theta\right)\right)} = A\left(\theta\right)B\left(\theta\right)C\left(\theta\right), \qquad \textit{for } \theta < \overline{\theta}, \tag{5.28}$$

[12] Hence, the taxes net of the social benefit given to the worst-off individuals $T\left(z_A\left(\theta\right)\right) - T\left(z_A\left(\underline{\theta}\right)\right)$ are reduced for everyone compared to the closed-economy ones.

where

$$A(\theta) := \frac{1 + e^M(\theta)}{e^H(\theta)},$$
$$B(\theta) := B_1(\theta) - B_2(\theta) - B_3(\theta),$$
$$C(\theta) := \frac{1 - F(\theta)}{\theta f(\theta)},$$

with

$$B_1(\theta) := \frac{1}{1 - F(\theta)} \int_\theta^{\overline{\theta}} \left[1 - \frac{\phi_\rho'(V_A(\tau))\, u_x'(x_A, z_A; \tau)}{\gamma}\right] \Psi_{\theta\tau} dF(\tau),$$
$$B_2(\theta) := \frac{1}{1 - F(\theta)} \int_\theta^{\overline{\theta}} \frac{\pi'(\tau)\, u_x'(x_A, z_A; \tau)}{\gamma} \Psi_{\theta\tau} d\tau, \qquad \pi'(\tau) \geq 0 \;\; (= 0 \text{ if } R(\tau) > 0),$$
$$B_3(\theta) := -\frac{1}{1 - F(\theta)} \frac{\iota(\overline{\theta})\, u_x'(x_A, z_A; \theta)}{\gamma}, \qquad \iota(\overline{\theta}) \geq 0 \;\; \left(= 0 \text{ if } R(\overline{\theta}) > 0\right),$$

where

$$\Psi_{\theta\tau} = \exp \int_\theta^\tau \left(1 - \frac{e^M(\delta)}{e^H(\delta)}\right) \frac{z_A'(\delta)}{z_A(\delta)} d\delta,$$

while $e^M(.)$ *and* $e^H(.)$ *are the Marshallian and Hicksian elasticities respectively. At the top,*

$$\frac{T'\left(z_A(\overline{\theta})\right)}{1 - T'\left(z_A(\overline{\theta})\right)} = \frac{A(\overline{\theta})}{\overline{\theta} f(\overline{\theta})} \frac{\iota(\overline{\theta})\, u_x'\left(x_A, z_A; \overline{\theta}\right)}{\gamma} \leq 0 \;\; \left(= 0 \text{ if } R(\overline{\theta}) > 0\right). \tag{5.29}$$

Proof. See 5.7.2 in the Appendix. ■

Proposition 5.5 extends Mirrlees's (1971) optimal income tax formula to take the threat of migration into account, using behavioural elasticities as in Saez (2001). It reflects the trade-off between efficiency and equity when the government has decided to maintain the national productive capacity to the maximum in preventing its citizens from leaving the country. $A(\theta)$ and $C(\theta)$ are the usual efficiency and demographic factors, respectively. However, the value of $A(\theta)$ is usually not the same whether the individuals can or cannot vote with their feet since it depends on gross income which is endogenous. The factor $B(\theta)$, which combines efficiency and equity, is the only factor which does not write as in Mirrlees's formula, in which the RHS of (5.28) reduces to $A(\theta) B_1(\theta) C(\theta)$. As previously stated, the optimal marginal tax rates can be strictly negative at the top, and therefore non-positive at interior points of the schedule.

Alternatively, $B_1(\theta) - B_2(\theta)$ can be written as

$$B_1(\theta) - B_2(\theta) = \int_\theta^{\overline{\theta}} [1 - g(\tau)] \Psi_{\theta\tau} dF(\tau), \tag{5.30}$$

where $g(\theta) = \left[\frac{\phi'_\rho(V_A(\theta))}{\gamma} + \frac{\pi'(\theta)}{\gamma f(\theta)}\right] u'_x(x_A, z_A; \theta)$ is the social marginal weight of the θ-individuals within the population. Higher social priority is thus given to the people threatening to emigrate. Since the optimal marginal tax rate at θ is inversely related to the aggregate social marginal weights of the individuals with greater productivity, we expect individual mobility to decrease the optimal marginal tax rates over a productivity range exceeding that where the participation constraints are binding. We now turn to the different channels captured in Formula (5.28) to look into this intuition.

As previously, we consider a small tax reform perturbation around the optimal income tax schedule. A small increase dT for gross income between z and $z + dz$ has *four* effects on social welfare. Three effects are already observed in closed economy and have been thoroughly examined by Saez (2001).

– *The three "usual" effects* allow us to grasp $A(\theta)$, $B_1(\theta)$ and $C(\theta)$.

First, the local increase in the marginal rate of tax mechanically results in individuals with gross income greater than z paying additional taxes. Second, the elasticity response from the taxpayers with gross income between z and $z + dz$ decreases their labour supply and reduces tax revenue. Third, under Assumption 5.2, the increase in taxes paid by these individuals has an income effect, leading them to work more, which is good for tax receipts.

– *The new participation effect* illuminates $B_2(\theta)$ and $B_3(\theta)$.

The tax reform perturbation *mechanically* results in an increase in taxes paid by all individuals with gross income strictly above z. Consequently, those among them for whom the participation constraints were already active receive now a utility level below their reservation utility. Then the participation constraints (PC) are no longer satisfied. So, these individuals have to be compensated for the increase in taxes they face.

We first examine the compensation for the individuals whose gross income is strictly below $z_{\overline{\theta}}$. The *substitution effect* leads A's government to *totally* compensate them for staying in A. Each of them is thus given $u'_x(x_A, z_A; \tau) \times dTdz$ additional units of utility. Since $\pi'(\tau)$ is the shadow price of the participation constraint at τ and γ the Lagrange multiplier of the tax revenue constraint (TR), the cost in terms of social welfare of the compensation of the τ-individuals amounts to

$$\frac{\pi'(\tau)\, u'_x(x_A, z_A; \tau)}{\gamma} \times dTdz. \tag{5.31}$$

The substitution effect combines with the usual *income effect*. Because leisure is a normal good under Assumption 5.2, the increase in the tax burden paid by all individuals with income greater than z induces them to work more. This allows A's government to increase the taxes they face. As a result, it is not required to compensate the potentially mobile individuals as high as the increase in taxes they face. We know from Saez (2001) that the magnitude of the uncompensated behavioural response is summarized by $\Psi_{\theta\tau} \geq 1$, which converts the social marginal utility of consumption of the τ-individuals, $u'_x(x_A, z_A; \tau)$, into that of the θ_z-individuals, $u'_x(x_A, z_A; \theta_z)$. Using (5.31), the social cost of the compensation of the τ-individuals, including income effects, is

$$\frac{\pi'(\tau)\, u'_x(x_A, z_A; \tau)}{\gamma} \Psi_{\theta\tau} \times dTdz. \tag{5.32}$$

Now, the overall social cost of compensating the individuals on the upper bound of the population is directly obtained as

$$-\frac{\partial W^N_{A,\rho} / \partial (V_B - c)\big|_{\overline{\theta}}}{\gamma} \times dTdz. \tag{5.33}$$

When the participation constraints are active at $\overline{\theta}$, $\partial W^N_{A,\rho} / \partial (V_B - c)\big|_{\overline{\theta}}$ is equal to $\partial W^N_{A,\rho} / \partial V_A\big|_{\overline{\theta}}$, which is $-\iota\left(\overline{\theta}\right)$. Converting (5.33) into social marginal utility of consumption at θ_z, one gets

$$\frac{\iota\left(\overline{\theta}\right) u'_x(x_A, z_A; \theta_z)}{\gamma} \times dTdz. \tag{5.34}$$

Finally, by (5.32) and (5.34), the average social cost of the compensation of all potentially mobile individuals with gross income above z is

$$\frac{1}{1 - F(\theta_z)} \left[\int_{\theta_z}^{\overline{\theta}} \frac{\pi'(\tau)\, u'_x(x_A, z_A; \tau)}{\gamma} \Psi_{\theta_z \tau} d\tau + \frac{\iota\left(\overline{\theta}\right) u'_x(x_A, z_A; \theta_z)}{\gamma} \right] \times dTdz$$
$$= \left[B_2(\theta_z) + B_3(\theta_z) \right] \times dTdz. \tag{5.35}$$

$B_2(\theta_z)$ is positive as soon as there are individuals with productivity above θ_z for whom the participation constraints are binding. This term goes therefore against progressivity on a range of gross income levels preceding that on which individuals hesitate to leave the country. This is because increasing the marginal tax rates at θ makes the compensation of all more productive individuals threatening to emigrate more expensive in terms of social welfare. In addition, for all $\theta < \theta^* : \pi'(\theta) = 0$, so

$$B_2(\theta) = \frac{1}{1 - F(\theta)} \int_{\theta^*}^{\overline{\theta}} \frac{\pi'(\tau)\, u'_x(x_A, z_A; \tau)}{\gamma} \Psi_{\theta^* \tau} d\tau. \tag{5.36}$$

Differentiating, one obtains

$$B'_2(\theta) = \frac{1 - F(\theta^*)}{(1 - F(\theta))^2} f(\theta)\, B_2(\theta^*), \tag{5.37}$$

which is strictly positive for $\theta \leq \theta^*$: the closer to θ^* the productivity level at which the small tax reform perturbation takes place, the higher the average compensation required to satisfy the participation constraints. When θ is greater than θ^*, it is not possible to determine the sign of $B_2(\theta)$ in the general case. Since $B_3(\theta)$ is non-negative, it reinforces the decrease in marginal tax rates induced by $B_2(\theta)$.

Eventually, the participation effect results in the adjustment of the optimal marginal tax rates to make the *average* tax rates compatible with the participation constraints. In consequence,

A's government should be particularly cautious about increasing marginal tax rates even at productivity levels where individuals do not hesitate to vote with their feet.

Diamond's Case

There has been considerable interest in the quasilinear-in-consumption version of Mirrlees model since the work by Diamond (1998) (cf. Atkinson (1990); Piketty (1997); Salanié (1998); d'Autume (2000); Boadway and Pestieau (2007); Saez (2001, 2002)). The absence of income effects on labour supply simplifies the optimal income tax analysis. In addition, it is not a too stringent assumption since most of the empirical studies give credence to small income effects relative to substitution effects as regards labour supply (Blundell, 1992; Blundell and MaCurdy, 1999). In our framework, another important implication of quasilinear-in-consumption preferences is that the migration costs are expressed in units of the consumption good. For the sake of clarity, we concentrate on the case where the elasticity of labour supply is constant. Taking stock of the results of Proposition 5.5 and Properties 5.2-5.3, the open-economy analogue of Diamond's formula reads as follows.

Proposition 5.6 *Let preferences be quasilinear-in-consumption and the elasticity of labour supply be constant, equal to e. Under the National criterion and in the absence of bunching, the optimal tax scheme has the following features :*
(i) for every θ which does not belong to an interval of positive length where (5.6) is active,

$$\frac{T'\left(z_A\left(\theta\right)\right)}{1-T'\left(z_A\left(\theta\right)\right)}=\frac{1+1/e}{\theta f\left(\theta\right)}\left[\int_{\theta}^{\overline{\theta}}\left(1-\frac{\phi_{\rho}'\left(V_A\left(\tau\right)\right)+\pi'\left(\tau\right)}{\gamma}\right)dF\left(\tau\right)-\iota\left(\overline{\theta}\right)\right] \tag{5.38}$$

with $\pi'(\tau)\geq 0$ $(=0$ if $R(\tau)>0)$;
(ii) for every θ which belongs to an interval of positive length where (5.6) is active,

$$T\left(z_A\left(\theta\right)\right)=\theta\left(\theta^e-c'\left(\theta\right)\right)\left[\theta^{\frac{e}{1+e}}\left(\theta^e-c'\left(\theta\right)\right)^{-\frac{1}{1+e}}-\frac{e}{1+e}\right]-\frac{\theta^{1+e}}{1+e}+c\left(\theta\right); \tag{5.39}$$

(iii) at junction points between (i) and (ii), (5.38) is equal to (5.21).

This restatement of Proposition 5.5 illuminates some aspects which are maybe left somewhat obscure in the general formulation. In particular, it makes clear that the structure of the solution basically consists of pieces of solution which are linked together. In the non-degenerate areas where the participation constraints are binding, the tax liability (5.39) intuitively corresponds to the consumption loss an individual would incur in case of migration. Outside those ares, the marginal tax rate formula is a generalization of Diamond's standard result. The characterization of the junction points between the pieces of solution exploits the fact that, in the absence of bunching, the marginal tax rate is herein a continuous function.

5.4.4. Citizen and Resident Criteria

Under the National criterion, the whole population is constrained to stay in A. We now relax this constraint to examine whether keeping everybody in the home country is not too expensive in terms of social welfare. For this purpose, we separate Problem 5.4 into two subproblems to determine the optimal $\widehat{\theta}$. In the first subproblem, $\widehat{\theta}$ is arbitrarily chosen by A's government.

Subproblem 5.1 *Given* $\widehat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$, *find* (x_A, z_A) *to maximize* $W^i_{A,\rho}\left(\widehat{\theta}\right)$, $i = \{C, R\}$, *subject to* $(FOIC)$, $(SOIC')$, (PC), (TR).

Let $\mathcal{W}^i_{A,\rho}\left(\widehat{\theta}\right)$ be the social value function of this subproblem, $\iota^i_{\widehat{\theta}}(\theta)$ the shadow price of incentive-compatibility constraint (FOIC), and $\pi^i_{\widehat{\theta}}(\theta)$ the shadow price of a uniform marginal increase in the reservation utility for all $\theta' \geq \theta$. The solution in $\widehat{\theta}$ to Problem 5.4 is then obtained as :

Subproblem 5.2 *Find* $\widehat{\theta}^i \in \left[\underline{\theta}, \overline{\theta}\right]$ *solution to* $\max_{\widehat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]} \mathcal{W}^i_{A,\rho}\left(\widehat{\theta}\right)$, $i = \{C, R\}$.

Subproblem 5.1 is a generalization of the second-best National problem where the upper productivity in A is exogenously given. Consequently, the optimal marginal tax rates share qualitative properties irrespective of the chosen social criterion. The only differences come from changes in the size of A's resident population.

Property 5.5 *Proposition 5.5 applies for the Citizen and Resident criteria provided :*
(i) $\overline{\theta}$ *is replaced by* $\widehat{\theta}^i$ *and* $1 - F(\theta)$ *by* $F\left(\widehat{\theta}^i\right) - F(\theta)$, $i = \{C, R\}$;
(ii) in $B_1(\theta)$, $\phi'_\rho(V_A)$ *is divided by* $F\left(\widehat{\theta}^R\right)$ *for the Resident criterion.*

Proof. See 5.7.3 in the Appendix. ■

We are now prepared to examine the allocation of individuals between A and B resulting from the implementation of the Citizen and Resident optimal income tax schedules. For all $\widehat{\theta} \in (\underline{\theta}, \overline{\theta})$, the $\widehat{\theta}$-individuals are indifferent between living in A or B. Let us assume $\widehat{\theta} < \overline{\theta}$. Hence, individuals with productivity above $\widehat{\theta}$ are in B. Making them relocate to A requires adjustments to prevent them from imitating less productive individuals. It also brings about a *new upward mimicking behaviour*. Indeed, A's residents can now have an incentive to mimicking them since they have the most appealing outside options.

The upward mimicking behaviour is crucial to understanding the interactions between the incentive-compatibility conditions and the type-dependent participation constraints. In closed economy, individuals have the usual incentive to understate their productivity θ to obtain greater social benefit whilst enjoying more leisure.[13] When type-dependent participation constraints are taken into account, the individuals may also be tempted to overstate his productivity θ, in working

[13] In the discrete population model of Guesnerie and Seade (1982), a sufficient condition for incentive-compatibility of the tax scheme is that only the downward adjacent incentive-compatibility constraints are binding (see also Weymark (1986b, 1987)). Hellwig (2007) has recently established, in both discrete and continuous models that under "desirability of redistribution" only the downward incentive-compatibility constraints are binding.

harder, to obtain greater compensation for staying in A. This behaviour reflects *countervailing incentives.* An asymmetry in terms of informational constraints between the individuals with productivities below $\widehat{\theta}$ and the $\widehat{\theta}$-individuals may therefore arise. Indeed, contrary to the former, the latter can only have the usual incentives. The cost of making the θ-individuals reveal their private information, represented by $\iota^i_{\widehat{\theta}}(\theta) \geq 0$, can thus have a downward jump discontinuity at $\widehat{\theta}$. However, making them reveal their private knowledge requires the gap between $V_B\left(\widehat{\theta}\right) - c\left(\widehat{\theta}\right)$ and $V_A(\theta)$ to be reduced. This increase in $V_A(\theta)$ reduces the social cost of a uniform increase in the reservation utility at θ and above, which is captured by $\pi(\theta)$. This effect stops suddenly when θ tends to $\widehat{\theta}$. Consequently, an *upward* jump discontinuity in $\pi^i_{\widehat{\theta}}$ corresponds to the *downward* jump discontinuity in $\iota^i_{\widehat{\theta}}$ at $\widehat{\theta}$. It turns out that these discontinuities have the same magnitude.

Property 5.6 *At* $\widehat{\theta}$,

$$\iota^i_{\widehat{\theta}}\left(\widehat{\theta}^-\right) - \iota^i_{\widehat{\theta}}\left(\widehat{\theta}\right) = \pi^i_{\widehat{\theta}}\left(\widehat{\theta}\right) - \pi^i_{\widehat{\theta}}\left(\widehat{\theta}^-\right) \geq 0 \left(= 0 \text{ if (PC) inactive at } \widehat{\theta}\right), \tag{5.40}$$

where $\iota^i_{\widehat{\theta}}\left(\widehat{\theta}^-\right) := \lim_{\theta \leq \widehat{\theta}} \iota^i_{\widehat{\theta}}(\theta)$.

Proof. See (5.82) in the Appendix. ■

A variational analysis provides insights into the costs and benefits of the presence in the home country of the marginal $\widehat{\theta}$-individuals.

Proposition 5.7 *Given* $\widehat{\theta} < \overline{\theta}$,

$$\frac{\partial \mathcal{W}^C_{A,\rho}\left(\widehat{\theta}\right)}{\partial \widehat{\theta}} = \gamma T\left(z_A\left(\widehat{\theta}\right)\right) f\left(\widehat{\theta}\right) + \iota\left(\widehat{\theta}^-\right) V'_A\left(\widehat{\theta}\right) - \left[\iota\left(\widehat{\theta}^-\right) - \iota\left(\widehat{\theta}\right)\right]\left|R'\left(\widehat{\theta}\right)\right|. \tag{5.41}$$

and

$$\frac{\partial \mathcal{W}^R_{A,\rho}\left(\widehat{\theta}\right)}{\partial \widehat{\theta}} = \frac{\gamma T\left(z_A\left(\widehat{\theta}\right)\right) f\left(\widehat{\theta}\right)}{F\left(\widehat{\theta}\right)} + \frac{\left[\phi_\rho\left(V_A\left(\widehat{\theta}\right)\right) - \mathcal{W}^R_{A,\rho}\left(\widehat{\theta}\right)\right] f\left(\widehat{\theta}\right)}{F\left(\widehat{\theta}\right)} + \frac{\iota\left(\widehat{\theta}^-\right) V'_A\left(\widehat{\theta}\right)}{F\left(\widehat{\theta}\right)} - \frac{\left[\iota\left(\widehat{\theta}^-\right) - \iota\left(\widehat{\theta}\right)\right]\left|R'\left(\widehat{\theta}\right)\right|}{F\left(\widehat{\theta}\right)}. \tag{5.42}$$

Proof. See 5.7.3 in the Appendix. ■

In (5.41) and (5.42), the first terms corresponds to the contribution of the marginal individuals to tax revenue. The second term in (5.42) compares the marginal and average social welfare. In contrast to the first-best, this term cannot be negative because (FOIC) ensures that the indirect utility is non-decreasing. The last two terms in (5.41) and (5.42) are specific to the second-best setting. They reflect the marginal costs and benefits with regard to incentives of the presence in A of the $\widehat{\theta}$-individuals.

First, there is a cost due to countervailing incentives. Indeed, individuals to the very left of $\widehat{\theta}$ have the possibility to mimic the $\widehat{\theta}$-individuals to benefit from their higher outside options. They can therefore claim an increase in their utility at the margin, equal to $V_B'\left(\widehat{\theta}\right) - c'\left(\widehat{\theta}\right) - V_A'\left(\widehat{\theta}\right) = \left|R'\left(\widehat{\theta}\right)\right|$. The shadow price of this upward-mimicking behaviour is given by the excess of $\iota\left(\widehat{\theta}^{-}\right)$ over $\iota\left(\widehat{\theta}\right)$, which is non-negative by Property 5.6. The corresponding marginal social cost is thus

$$\left[\iota\left(\widehat{\theta}^{-}\right) - \iota\left(\widehat{\theta}\right)\right]\left|R'\left(\widehat{\theta}\right)\right| \geq 0. \tag{5.43}$$

If $\widehat{\theta}$ is a non-isolated point where (PC) is active, this cost is nil because $R'\left(\widehat{\theta}\right) = 0$.

Second, when countervailing incentives arise, the individuals to the very left of $\widehat{\theta}$ have greater utility. They thus are less inclined to mimic less productive individuals. The slope of the indirect utility V_A' at $\widehat{\theta}$ required for them to reveal their type truthfully is therefore reduced at the margin. Since $\iota\left(\widehat{\theta}^{-}\right)$ is the shadow price of (FOIC), the marginal social benefit of this slackening of the downward incentive compatibility constraints is $\iota\left(\widehat{\theta}^{-}\right)V_A'$. This is the second implication of countervailing incentives due to the presence of the marginal individuals. Finally, the *net* marginal social cost incurred to restore incentives at the top amounts to

$$\left[\iota\left(\widehat{\theta}^{-}\right) - \iota\left(\widehat{\theta}\right)\right]\left|R'\left(\widehat{\theta}\right)\right| - \iota\left(\widehat{\theta}^{-}\right)V_A'\left(\widehat{\theta}\right). \tag{5.44}$$

In the light of Proposition 5.7, the marginal individuals positively contribute to social welfare when (i) they pay positive taxes and (ii) either ι is continuous at $\widehat{\theta}$ or $\widehat{\theta}$ is a non-isolated point where (5.6) is active. These situations correspond to cases where the usual downward mimicking behaviour predominates for highly skilled individuals. However, they do not exhaust all possible cases. In particular, the trade-off between maintaining national capacity to the maximum and sustaining the redistribution programme become more complex when $\widehat{\theta}$ is an isolated point where (5.6) is active and ι has a jump discontinuity. In the solution to 5.2, $\partial\mathcal{W}_{A,\rho}^{i}\left(\widehat{\theta}\right)/\partial\widehat{\theta}$ must be non-negative, $i = \{C, R\}$. Otherwise, emigration of the most productive individuals would increase social welfare. This leads to the following sufficient condition under which emigration of the most productive individuals initially living in A is socially optimal.

Proposition 5.8 *Consider the National optimal allocation. Assume $\overline{\theta}$ is an isolated point where (PC) is active and ι has a jump discontinuity. Then,*
(i) $\widehat{\theta}^{C} < \overline{\theta}$ if

$$\gamma T\left(z_A\right) f < \left[\iota\left(\overline{\theta}^{-}\right) - \iota\right]\left|R'\right| - \iota\left(\overline{\theta}^{-}\right)V_A'; \tag{5.45}$$

(ii) $\widehat{\theta}^{R} < \overline{\theta}$ if

$$\gamma T\left(z_A\right) f + \left[\phi_\rho\left(V_B - c\right) - W_{A,\rho}^{N}\right] f < \left[\iota\left(\overline{\theta}^{-}\right) - \iota\right]\left|R'\right| - \iota\left(\overline{\theta}^{-}\right)V_A', \tag{5.46}$$

where all functions are evaluated at $\overline{\theta}$ except otherwise stated.

Proof. See 5.7.3 in the Appendix. ■

This proposition exploits the fact that both Citizen and Resident criteria coincide with the National one when $\widehat{\theta} = \overline{\theta}$. It is then possible to apply the previous cost/benefit analysis to the National criterion. (5.45) and (5.46) correspond to cases where $\partial \mathcal{W}^i_{A,\rho}\left(\widehat{\theta}\right)/\partial\widehat{\theta}_{\widehat{\theta}=\overline{\theta}} < 0$, $i = \{C, R\}$.

The basic intuition behind Proposition 5.8 is that the choice of $\widehat{\theta}$ by A's government can be regarded as a means of revealing private information. Indeed, if A's government designs a tax policy such that the individuals with productivity greater than $\widehat{\theta}$ do not receive in A their reservation utility, it *knows* that $\widehat{\theta}$ is the maximum productivity in its resident population and consequently that the individuals with productivity greater than $\widehat{\theta}$ are in B. Proposition 5.8 tells us in which cases using this means improves social welfare.

5.5. NUMERICAL RESULTS

We already know that individual mobility is harmful to progressivity and significantly alters the qualitative properties of the optimal non-linear income tax schedule. It remains to *quantify* the magnitude of the changes with respect to the Mirrleesian closed economy model. In particular, we would like to examine whether potential mobility of a few highly skilled individuals has more than a negligible effect on the optimal policy. For this purpose, we adopt the National criterion and calibrate A's economy to roughly correspond to the French one.

5.5.1. Calibration

We employ a truncated lognormal distribution for the lower part of the productivity distribution and complete it with a Pareto tail with density $f(\theta) = K/\theta^{1+a}$. The two parameters of the lognormal productivity distribution distribution are obtained by inversion from the French survey data "Budget des familles", year 1995, as portrayed in Figure 4 in Laslier et al. (2003). We get a mean of 0.2398 and a variance of 0.4403[14]. Following Hungerbühler et al. (2005), we take $a = 2$, choose K and the boundary between both distributions in such a way that the entire distribution is continuously differentiable. We normalize the productivity levels so that the median individuals have productivity equal to the median income in 1995, i.e. 13 320 euros. The productivity support is the positive real line with an upper bound equal to 15 times the median productivity.

We focus on the case where there are no income effects on labour supply and e^H is constant, with $e^H = 0.2$, as in d'Autume (2000). Preferences are thus given by (5.17). For convenience, the government is assumed to be Rawlsian.

Migration costs are the new ingredient of our model. They correspond to all the costs an individual will have to pay because of his choice of migration. Since the model is static, these

[14] Since our model does not take the family size into account, the population is restricted to single individuals.

costs as well as the utility levels should be regarded as expected values. Very few empirical work have studied the individual costs of migration[15]. We use constant costs as a benchmark and calibrate them so as to reflect plausible scenarios as regards the proportion of individuals threatening to emigrate : 10%, 5%, 3%, 1%, 0.5% and 0.1%.[16] We obtain migration costs equal to 15 550, 27 900, 40 500, 77 400, 104 300 and 151 900 euros per annum respectively. We also provide simulations for linearly decreasing costs. The highest skilled individuals have migration costs equal to 40 500 and 77 400 euros, values used in the constant case. The costs are then linearly adjusted to obtain threat of migration by 1% and 0.5% respectively.

5.5.2. Numerical Results

Figures 5.2–5.4 and Table 5.1 in Appendix A.4 contrast the second-best optimal allocations for constant migration costs in the six scenarios described above. For instance, when 3% of the population threaten to emigrate, the social welfare, equal to the redistributive budget under the maximin, is reduced by 5.8%. The individuals paying the maximum average tax have gross income $\widehat{z}_A = 70\ 834$€/year. The optimal average tax rates are decreasing above this level, even if the participation constraints are only active for individuals with gross income above $z_A^* = 98\ 779$€/year. The range of decrease corresponds to 4.4% of the population. Even if the individual and social utility levels do only slightly vary compared to the benchmark, Figure 5.2 emphasizes that the changes in the tax schedule are very noticeable, even when the proportion of potentially mobile individuals is very low.

Specifically, even if the average tax rate profile is already single-peaked in closed-economy, the corresponding graphs are far more hump-shaped when the threat of migration goes up (Figure 5.2.b). The lower bound $\widehat{z}_A$ of the range of gross income from which the average tax rate is decreasing (cf. the black circles) is smaller than the gross income z_A^* from which the participation constraints are active (cf. the squares). Interestingly, the smaller the proportion of the population threatening to emigrate, the larger the gap between $\widehat{z}_A$ and z_A^* as well as the ratio

$$1 + \frac{1 - F\left(\theta_{z_A^*}\right)}{1 - F\left(\theta_{\widehat{z}_A}\right)}. \tag{5.47}$$

The latter is approximately equal to 1.26, 1.34, 1.47, 2, 2.6, 7 for potential emigration by 10%, 5%, 3%, 1%, 0.5%, and 0.1% of the population. In this respect, the threat of migration seems to have a multiplicative power all the stronger as fewer people would like to emigrate. It is really a feature that only simulations may reveal.

Figure 5.3 and 5.4 contrast the open and closed optimal allocations from a distributional

[15] For instance, the IZA Database for Migration Literature provides 34 matches for "moving costs" (http ://www.iza.org/iza/en/webcontent/links/migration). These references are mainly theoretical or estimate the macroeconomic costs of migration.

[16] In pratice, we have solved the optimal income tax problem and compute the proportion of the population on the participation constraints assuming constant migration costs $c(\theta) = 50, 75, 100, \cdots, 200000$ euros. Then, we have chosen those values of $c(\theta)$ for which 10%, 5%, 3%, 1%, 0.5% and 1% of the population threaten to emigrate.

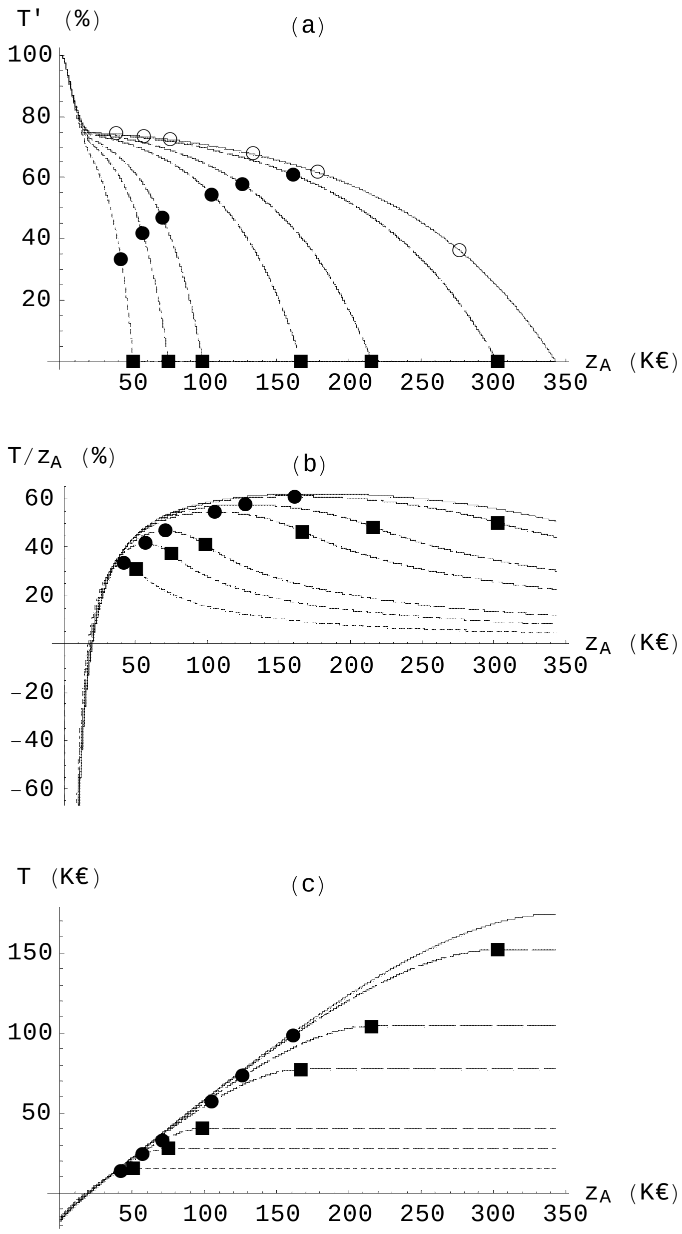

FIG. 5.2 – Constant Migration Costs. (a) Optimal Marginal Tax Rates ; (b) Optimal Average Tax Rates ; (c) Optimal Taxes. The solid line refers to the closed economy benchmark. Otherwise, the less dotted the line, the lower the threat of migration : 10% ; 5% ; 3% ; 1% ; 0.5% and 0.1% respectively. Squares correspond to $\widehat{z}_A$; black circles to z_A^* ; empty circles to the choice the θ^*-individuals would make if the economy were closed.

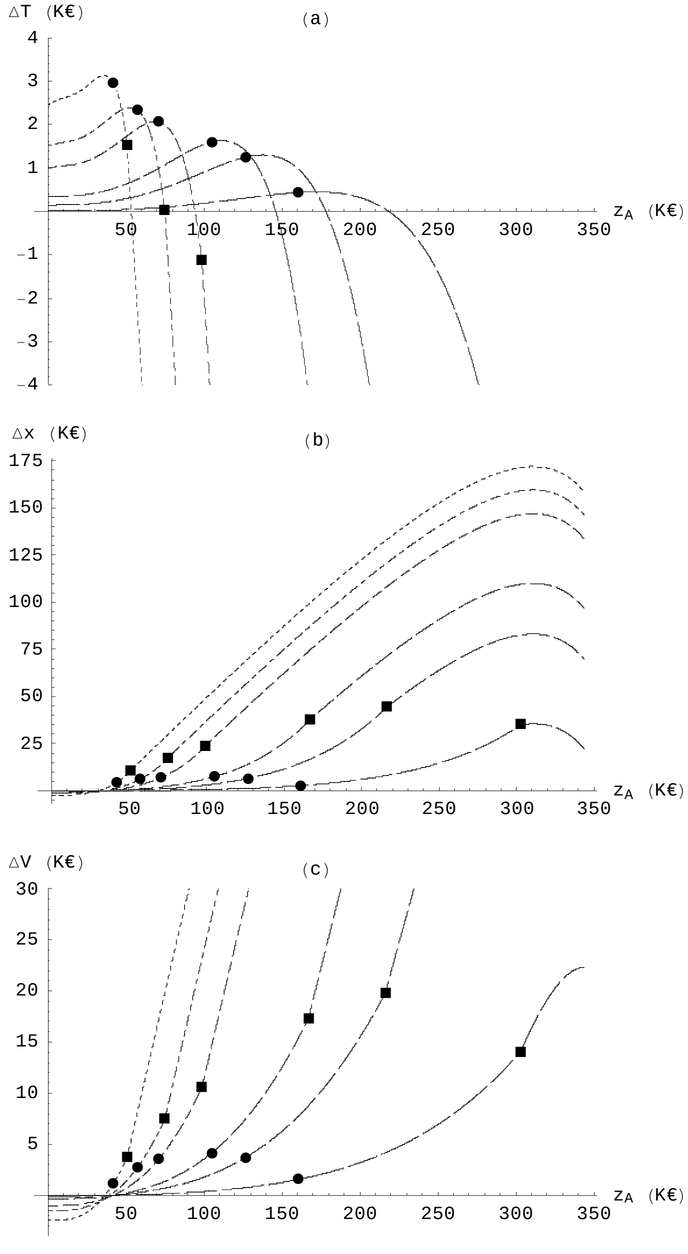

FIG. 5.3 – Constant Migration Costs. Increase w.r.t closed economy benchmark : (a) Taxes ; (b) Consumption ; (c) Utility. The less dotted the line, the lower the threat of migration : 10% ; 5% ; 3% ; 1% ; 0.5%, 0.1% respectively. Squares correspond to $\widehat{z}_A$; circles to z_A^*.

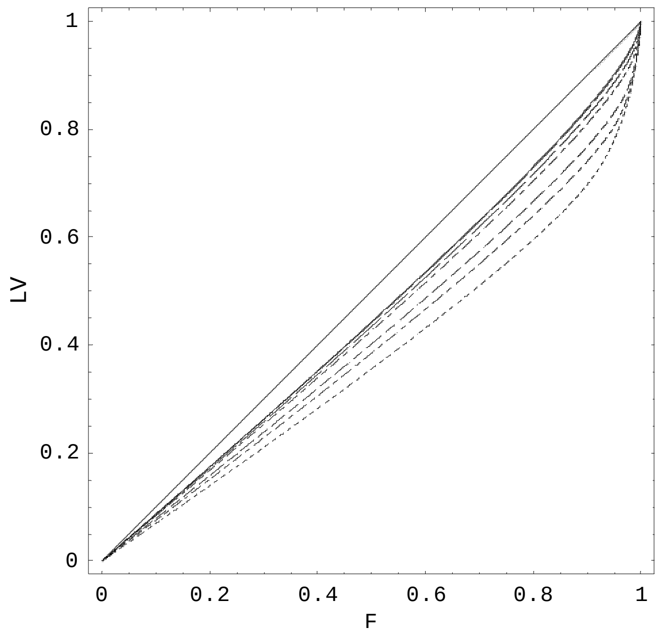

FIG. 5.4 – Constant Migration Costs. Lorenz Curves for the Indirect Utility Levels. The less dotted the line, the lower the threat of migration : 10% ; 5% ; 3% ; 1% ; 0.5%, 0.1% respectively. The solid line below the 45° -line pertains to the closed economy benchmark.

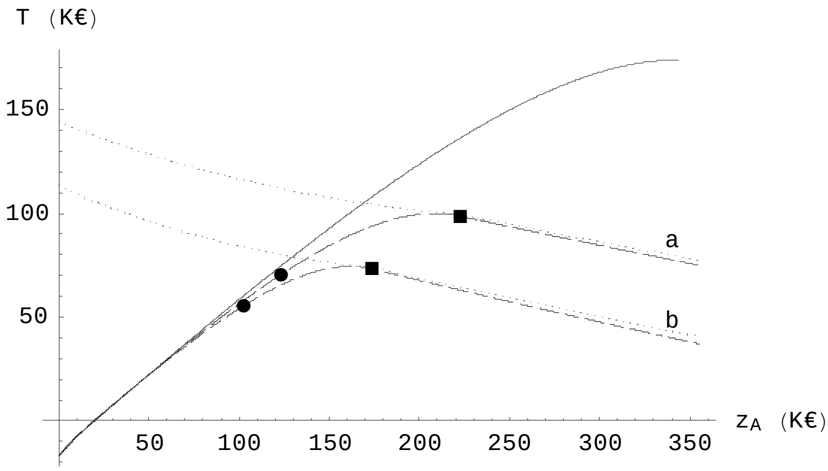

FIG. 5.5 – Decreasing Migration Costs. The dotted lines give the migration costs ; the dashed lines the optimal tax liabilities. The less dashed the latter, the greater the threat of migration : (a) 0.5% ; (b) 1%. Squares correspond to $\widehat{z}_A$; circles to z_A^*. The solid line pertains to the closed economy benchmark.

point of view[17]. The highly skilled appear as the real winners, since they pay less taxes (Figure 5.3.a) and have higher utility (Figure 5.3.c). The situation of the low-skilled does not worsen as much as one could expect. Individuals with gross income close to $\widehat{z}_A$ are actually the real losers in terms of taxes. Nevertheless, they slightly benefit from the openness of the economy in terms of utility. In fact, the decline in marginal tax rates allow them to increase their gross and net income sufficiently to overbalance the resulting loss in leisure (Figure 5.3.b). Consequently, the deterioration of the middle-skilled workers' situation in terms of taxes does not translate into losses in individual welfare as observed in the first-best. In spite of this rather comforting result, inequality of utilities deepens (Figure 5.4).

Figure 5.5 provides examples of optimal tax schemes for decreasing migration costs[18]. The tax liabilities are hump-shaped so that the middle-skilled have actually to pay greater taxes than the highly-skilled. In contrast to the case where $c(\theta)$ is constant, people on their participation constraints pay lower taxes than their migration costs. In the first-best setting, they would be taxed as high as their migration costs. Therefore, taking incentive compatibility into account restricts the tax levy on the potentially mobile individuals.

5.6. CONCLUSION

This paper provides a first example of the introduction of type-dependent participation constraints in the optimal income tax framework. These constraints interact with the standard constraints in a non-trivial way and make the structure of the mimicking behaviour more complex than in closed economy. Since they induce substantial changes, it might be worth introducing them in other classic models of taxation theory, like those devoted to capital taxation.

In this extended framework, the issue of the optimal allocation of individuals between the home country and abroad is embedded in the optimal income tax problem. Consequently, a new trade-off between maintaining the redistribution programme and preserving national productive capacities adds to the traditional trade-off between equity and efficiency. In the first-best, emigration of highly skilled individuals should always be prevented, except from the resident criterion. In the second-best, this may be false because of countervailing incentives.

Key qualitative features of the optimal income tax policy obtained in closed economy do no longer hold. The participation effect does not only favour a decrease in the optimal marginal tax rates; it can also make them strictly negative. Consequently, the optimal average tax rates as well as the optimal tax liabilities can be decreasing. Numerical simulations show that the threat of migration has a significant impact even when the proportion of potentially mobile individuals is very low. They also reveal that if the highly skilled are the real winners, the welfare of the low-skilled is only slightly reduced because quite high taxes can still be levied on the middle-skilled.

[17] The gross income received by the θ-individuals depends on the tax schedule itself. For instance, we have represented the choice of the θ^*-individuals if the economy were closed by empty circles in Figure 5.2a. As a matter of fact, the variation in taxes in Figure 5.3.a is not obtained as the difference between taxes paid with and without mobility for a given *gross income* that can be read in Figure 5.2.c.

[18] Linearity in θ does not imply linearity in z_A, as shown by the dotted lines.

Our qualitative and quantitative results convey a curse of the middle-skilled workers : this curse is expressed in terms of utility in the first-best, and in terms of average tax rates and tax liabilities in the second-best.

The backbone of the analysis remains valid even when B's government implements a non-linear income tax policy, provided this policy is given. Consequently, the material of this paper paves the way for deriving the reaction function of A to B's tax policy and vice versa. It thus provides the basic ingredients of a symmetric game on redistributive non-linear income taxes, the solution of which is left for further research.

5.7. APPENDIX

5.7.1. First-Best

Proof of Proposition 5.1. Let π' and γ be the Lagrange multipliers of (PC) and (TR) respectively. Under Assumption 5.1, the solution is interior and the SOC are satisfied. Hence, the necessary and sufficient FOC are

$$\left(\phi'_\rho + \pi'\right) U'_x = \gamma \text{ and } \left(\phi'_\rho + \pi'\right) U'_\ell = -\gamma\theta, \tag{5.48}$$

with

$$\pi' \geq 0,\ U(x,\ell) - V_B + c \geq 0,\ \pi' \left[U(x,\ell) - V_B + c\right] = 0,\ \forall\theta \in \left[\underline{\theta}, \overline{\theta}\right]. \tag{5.49}$$

Since $\phi'_\rho > 0$, (5.48) and (5.49) implies $\gamma > 0$. The following Lemma is shown in Mirrlees (1974) for $\rho = 0$.

Lemma 5.1 *Let J be a non-empty open interval where $\pi' \equiv 0$. Then for all $\theta \in J$, (a) $V'_A(\theta) < 0$ when $0 \leq \rho < \infty$, (b) $V'_A(\theta) = 0$ when $\rho \to \infty$.*

Proof. Assumption 5.2 holds if and only if $d\ell/dT > 0$. Since $\pi' \equiv 0$, applying the implicit function theorem to (5.48) yields

$$\frac{d\ell(\theta)}{dT(\theta)} = -\frac{\theta U''_{xx} + U''_{x\ell}}{\theta\left[2U''_{x\ell} - \frac{U'_\ell}{U'_x}U''_{xx} - \frac{U'_x}{U'_\ell}U''_{\ell\ell}\right]}, \tag{5.50}$$

where the square bracket is strictly positive because U is strictly quasi-concave under Assumption 5.1. Therefore, Assumption 5.2 is equivalent to

$$\theta U''_{xx} + U''_{x\ell} < 0. \tag{5.51}$$

(a) Since $\pi' \equiv 0$, (5.48) yields $\theta = U'_x/U'_\ell$ and, by differentiation,

$$\begin{pmatrix} U''_{xx} - \gamma\rho U^{\rho-1}U'_x & U''_{x\ell} - \gamma\rho U^{\rho-1}U'_\ell \\ U''_{x\ell} + \theta\gamma\rho U^{\rho-1}U'_x & U''_{\ell\ell} + \theta\gamma\rho U^{\rho-1}U'_\ell \end{pmatrix} \begin{pmatrix} x'(\theta) \\ \ell'(\theta) \end{pmatrix} = \begin{pmatrix} 0 \\ -\gamma U^\rho \end{pmatrix}. \tag{5.52}$$

As $|A| > 0$ under Assumption 5.1,

$$\begin{cases} x'(\theta) = \gamma U^{\rho}\left[U''_{x\ell} - \gamma\rho U^{\rho-1} U'_{\ell}\right] / |A| \\ l'(\theta) = -\gamma U^{\rho}\left[U''_{xx} - \gamma\rho U^{\rho-1} U'_{x}\right] / |A| \end{cases} \tag{5.53}$$

from which

$$V'_A(\theta) = u'_x x'(\theta) + u'_{\ell}\ell'(\theta) = -\gamma U^{\rho} U'_{\ell}\left[U''_{xx} - U''_{x\ell}U'_x/U'_{\ell}\right] / |A|, \tag{5.54}$$

which has the same sign as $U''_{xx} - U''_{x\ell}U'_x/U'_{\ell}$, i.e. as $\theta U''_{xx} - U''_{x\ell}$. Hence, by (5.51), $V'_A(\theta) < 0$.
(b) The result directly follows from duality. ■

Step 1 : The existence of θ^* is obvious. Indeed, since $V_A^{c\ell}\left(\overline{\theta}\right) < V_B\left(\overline{\theta}\right) - c\left(\overline{\theta}\right)$, the closed-economy solution violates (5.6) ; so there are θ such that $\pi' > 0$ at the solution to Problem 5.1.

Step 2 : $\pi'(\theta) > 0$ for all $\theta > \theta^*$.

By (5.48), $\pi'(\theta) = \gamma/U'_x - \phi'_{\rho}$, which implies under Assumption 5.1 and the continuity of T, the continuity of π'. Assume $\theta' := \min\left\{\theta \in \left[\theta^*, \overline{\theta}\right] : \pi'(\theta) = 0\right\}$ exists. Then, by continuity of π', there exists $\theta'' > \theta'$ such that $\pi' = 0$ on $\left[\theta', \theta''\right]$. By continuity of R, $R\left(\theta'\right) = 0$. On $\left[\theta', \theta''\right]$, $V'_A \leq 0$ by Lemma 5.1 and $V'_B - c' > 0$ under Assumption 5.3. Then $R < 0$ for $\theta \in \left(\theta', \theta''\right)$, contradicting (PC). Hence, θ' does not exist. ■

5.7.2. Second-Best : National Criterion

Proof of Proposition 5.4. Since $V_A = \theta\ell_A - T - v(\ell_A)$, by 5.17, the government chooses ℓ_A and V_A to maximize

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta\ell_A(\theta) - V_A(\theta) - v(\ell_A(\theta))\right] dF(\theta) \text{ s.t. (FOIC) and (PC).}$$

ℓ_A is control variable ; V_A state variable with adjoint variable ι. The Hamiltonian and Lagrangian are

$$H^N = \left[\theta\ell_A - V_A - v(\ell_A) + \iota\frac{\ell_A}{\theta}v'(\ell_A)\right] f, \tag{5.55}$$
$$L^N = H^N + \pi' R.$$

We call $\{\theta_j\}_{j=1}^N$ points where (5.6) becomes or ceases to be active as well as $\underline{\theta}$ and $\overline{\theta}$. By Theorem 2 in Seierstad and Sydsaeter (1987, p. 332-335)[19,20], necessary conditions are :

$$\partial H^N/\partial \ell_A = 0 \Leftrightarrow (\theta - v')\, f + \iota \left(\frac{v'}{\theta} + \frac{\ell_A}{\theta} v''\right) = 0, \tag{5.56}$$

$$\partial L^N/\partial V_A = -\iota'(\theta) \Leftrightarrow \iota'(\theta) = f - \pi', \tag{5.57}$$

$$\iota\left(\overline{\theta}\right) \geq 0 \ \left(= 0 \text{ when } R\left(\overline{\theta}\right) > 0\right), \tag{5.58}$$

$$\iota\left(\underline{\theta}\right) \leq 0 \ \left(= 0 \text{ when } R\left(\underline{\theta}\right) > 0\right), \tag{5.59}$$

$$\pi'(\theta) \geq 0,\ R(\theta) \geq 0,\ \pi'(\theta) R(\theta) = 0, \tag{5.60}$$

$$\iota\left(\theta_j^-\right) - \iota\left(\theta_j^+\right) = \pi\left(\theta_j^+\right) - \pi\left(\theta_j^-\right) \geq 0 \ (= 0 \text{ if } R(\theta_j) > 0). \tag{5.61}$$

Since $v'/\theta = 1 - T'$ and $e^H(\theta) = v'/(\ell_A v'')$, rearranging (5.56) yields

$$\frac{T'}{1-T'} = -\frac{\iota}{\theta f}\left(1 + \frac{1}{e^H(\theta)}\right). \tag{5.62}$$

ι and π' are piecewise continuous and thus integrable. Integration of (5.57) between θ and $\overline{\theta}$ gives

$$\iota(\theta) = \iota\left(\overline{\theta}\right) - \int_\theta^{\overline{\theta}} \iota'(\theta)\, d\theta = \iota\left(\overline{\theta}\right) - \int_\theta^{\overline{\theta}} [f - \pi']\, d\theta = \iota\left(\overline{\theta}\right) - 1 + F(\theta) + \int_\theta^{\overline{\theta}} \pi' d\theta, \tag{5.63}$$

which is plugged into (5.62). The proof is completed in two steps.

Step 1 : $\int_\theta^{\overline{\theta}} \pi' d\theta = \int_{\theta \in [\theta, \overline{\theta}] \cap \Theta^{PC}} f(\theta)\, d\theta = \mu_f\left(\left[\theta, \overline{\theta}\right] \cap \Theta^{PC}\right)$.

Consider any non-empty open interval in $\left[\underline{\theta}, \overline{\theta}\right]$ where $\pi' > 0$. By Property 5.4, $T' = 0$. Hence, for all θ in this interval $\iota(\theta) = 0$ by (5.62) and thus $\iota'(\theta) = 0$; so by (5.57), $\pi'(\theta) = f(\theta)$. Since (PC) is only active on non-degenerate intervals, the equality above is obtained.

Step 2 : $\iota\left(\overline{\theta}\right) = 0$.

If (PC) is inactive at $\overline{\theta}$, $\iota\left(\overline{\theta}\right) = 0$ because of (5.58). Otherwise, (PC) is active at $\overline{\theta}$ and, under assumptions, there is $\varepsilon > 0$ such that (PC) is active on $I = (\overline{\theta} - \epsilon, \overline{\theta}]$. Since e^H is constant, by Property 5.4, $T' = 0$ on I. Hence, $\iota\left(\overline{\theta}\right) = 0$ by (5.62). ■

Proof of Proposition 5.5. z_A is control variable; V_A and $G(\theta) := \int_{\underline{\theta}}^{\theta} T(z_A(\tau))\, dF(\tau)$ are state variables. Since $T := z_A - x_A$, Leibnitz's rule yields $G'(\theta) = (z_A(\theta) - x_A(\theta))\, f(\theta)$. The isoperimetric constraint (TR) is taken into account through G' and the boundary conditions $G(\underline{\theta}) = 0$ and $G\left(\overline{\theta}\right) = 0$. It is not necessary to take x_A explicitly into account because it is uniquely determined by V_A and z_A. Let $x_A = h(V_A, z_A; \theta)$; differentiating shows $\partial x_A/\partial V_A =$

[19] The Theorem we referred to is applied as follows. Since the adjoint variables are assumed to have a finite number of jump discontinuities and be $\mathcal{C}^1$ elsewhere, the "almost necessary conditions" (p. 335) are in fact necessary. In the Theorem, $q' = \lambda$ is our π'. Hence, q is our π and λ our π'. Consequently, their $\beta^k = \pi\left(\tau_k^+\right) - \pi\left(\tau_k^-\right)$. We then employ their Eq. (5.37) to get (5.61).

[20] The necessary conditions are often stated for state variables which are fixed at the initial point, which is not the case presently. We have used Seierstad and Sydsaeter (1987, Theorem 5, pp. 185, Eq. 30b) (referred to as (S-S) from now on) to obtain (5.59). This remark applies to the other proofs in the paper.

$1/u'_x$ and $\partial x_A/\partial z_A = s$. The Hamiltonian and Lagrangian are respectively

$$\begin{aligned} H^N &= \phi_\rho\left(V_A\right) f + \iota u'_\theta + \gamma\left(z_A - x_A\right) f, \\ L^N &= H^N + \pi' R. \end{aligned}$$

As $\partial u'_\theta/\partial z_A = u''_{\theta z} + s u''_{\theta x} = -u'_x s'_\theta$, and $\partial u'_\theta/\partial V_A = u''_{\theta x}/u'_x$, necessary conditions are :

$$\partial H^N/\partial z_A = 0 \Leftrightarrow \iota u'_x s'_\theta - \gamma\left(1-s\right) f = 0, \tag{5.64}$$

$$\partial L^N/\partial V_A = -\iota' \Leftrightarrow \iota'\left(\theta\right) = -\phi'_\rho\left(V_A\right) f - \iota u''_{\theta x}/u'_x - \pi' + \gamma f/u'_x, \tag{5.65}$$

$$\partial L^N/\partial G = -\gamma' \Leftrightarrow \gamma' = 0, \tag{5.66}$$

$$(5.58),\ \ (5.59)\,,\ (5.60),\ \ (5.61)\,. \tag{5.67}$$

$\gamma\left(\theta\right)$ is constant, equal to $\gamma > 0$. As $s = 1 - T'$, $T' = \iota u'_x s'_\theta/\left(\gamma f\right)$ by (5.64). In addition, using basic calculus, $\left[1 + e^M\left(\theta\right)\right]/e^H\left(\theta\right) = -\theta s'_\theta/s$. Hence,

$$\frac{T'}{1-T'} = -\frac{\iota u'_x}{\gamma\theta f}\frac{1+e^M\left(\theta\right)}{e^H\left(\theta\right)}. \tag{5.68}$$

When $\theta = \overline{\theta}$, (5.68) and (5.58) yield (5.29). When $\theta < \overline{\theta}$, (5.68) can be rewritten as

$$\frac{T'}{1-T'} = -\frac{\iota u'_x}{\gamma\left(1-F\left(\theta\right)\right)}\frac{1+e^M\left(\theta\right)}{e^H\left(\theta\right)}\frac{1-F\left(\theta\right)}{\theta f\left(\theta\right)}, \tag{5.69}$$

If $(.;\tau)$ means evaluation at $\left(x_A\left(\tau\right), z_A\left(\tau\right);\tau\right)$, integrating (5.65) between θ and $\overline{\theta}$ yields

$$\iota\left(\theta\right) = \iota\left(\overline{\theta}\right) + \int_\theta^{\overline{\theta}}\left(\phi'_\rho\left(V_A\left(\tau\right)\right) f\left(\tau\right) + \pi'\left(\tau\right) - \frac{\gamma f\left(\tau\right)}{u'_x\left(.;\tau\right)}\right)\widetilde{\Psi}_{\theta\tau} d\tau, \tag{5.70}$$

with $\widetilde{\Psi}_{\theta\tau} := \exp\int_\theta^\tau u''_{\theta x}\left(.;\tau'\right)/u'_x\left(.;\tau'\right) d\tau'$. The following relation has been proved by Saez (2001, p. 227) :

$$\Psi_{\theta\tau} := \frac{u'_x\left(.;\theta\right)}{u'_x\left(.;\tau\right)}\widetilde{\Psi}_{\theta\tau} = \exp\int_\theta^\tau\left(1 - \frac{e^M\left(\tau'\right)}{e^H\left(\tau'\right)}\right)\frac{z'_A\left(\tau'\right)}{z_A\left(\tau'\right)} d\tau'. \tag{5.71}$$

Using (5.70) and (5.71),

$$-\frac{\iota\left(\theta\right) u'_x\left(.;\theta\right)}{\gamma} = \int_\theta^{\overline{\theta}}\left[1 - \left(\phi'_\rho\left(V_A\left(\tau\right)\right) + \frac{\pi'\left(\tau\right)}{f\left(\theta\right)}\right)\frac{u'_x\left(.;\tau\right)}{\gamma}\right]\Psi_{\theta\tau} dF\left(\tau\right) - \frac{\iota\left(\overline{\theta}\right) u'_x\left(.;\theta\right)}{\gamma}, \tag{5.72}$$

and plug the obtained expression in (5.69). ■

5.7.3. Second-Best : Citizen and Resident Criteria

Proof of Property 5.5. (a) *Citizen criterion.* By definition, $W^C_{A,\rho}\left(\widehat{\theta}\right)$ is maximum when $\widehat{\theta} = \widehat{\theta}^C$, i.e. when $W^C_{A,\rho}\left(\widehat{\theta}^C\right)$ is maximized with respect to (x_A, z_A) subject to (FOIC), (PC), (TR). The FOC are the same as (5.64)–(5.67), except that $\overline{\theta}$ is replaced by $\widehat{\theta}^C$. We then proceed as in the proof of Proposition 5.5.

(b) *Resident criterion.* By definition, $W^R_{A,\rho}\left(\widehat{\theta}\right)$ is maximum when $\widehat{\theta} = \widehat{\theta}^R$, i.e. when $W^R_{A,\rho}\left(\widehat{\theta}^R\right)$ is maximized with respect to (x_A, z_A) subject to (FOIC), (PC), (TR). The FOC are the same as (5.64)–(5.67), except that (i) $\overline{\theta}$ is replaced by $\widehat{\theta}^R$ and (ii) $\phi'_\rho(V_A)$ is divided by $F\left(\widehat{\theta}^R\right)$. We then proceed as in the proof of Proposition 5.5. ■

Proof of Proposition 5.7. We proceed in two steps.

Step 1 : We first state necessary conditions for a maximum in Subproblem 5.1. These conditions are the same under the National and Resident criteria since $\widehat{\theta}$ is given. $\zeta_A := z'_A$ is control variable ; z_A, V_A and G are state variables ; η, ι and γ are adjoint variables. (SOIC) is transformed into $g(\zeta_A) \geq 0$ to avoid dealing with singular solutions, where g is a $\mathcal{C}^2$-function such that $g' > 0$ and $g(0) = 0$. The Hamiltonian and Lagrangian are

$$\begin{aligned} H^i &= \phi_\rho(V_A) f + \eta\zeta_A + \iota u'_\theta + \gamma(z_A - x_A) f, \\ L^i &= H^R + \pi' R + \kappa g(\zeta_A), \end{aligned}$$

with $i = \{N, R\}$. A solution to Subproblem 5.1 must satisfy :

$$\partial L^i/\partial\zeta_A = 0 \Leftrightarrow \eta + \kappa g'(\zeta_A) = 0, \tag{5.73}$$

$$\eta' = -\partial L^i/\partial z_A \Leftrightarrow \eta' = \iota u'_x s'_\theta - \gamma(1-s) f, \tag{5.74}$$

$$\iota' = -\partial L^i/\partial V_A \Leftrightarrow \iota' = -\phi'_\rho f - \iota u''_{\theta x}/u'_x + \gamma f/u'_x - \pi', \tag{5.75}$$

$$\gamma' = -\partial L^i/\partial G \Leftrightarrow \gamma' = 0, \tag{5.76}$$

$$\pi' \geq 0,\ R \geq 0,\ \pi' R = 0, \tag{5.77}$$

$$\kappa \geq 0,\ g(\zeta_A) \geq 0,\ \kappa g(\zeta_A) = 0, \tag{5.78}$$

$$\eta(\underline{\theta}) = \eta\left(\widehat{\theta}\right) = 0, \tag{5.79}$$

$$\iota(\underline{\theta}) \leq 0\ \ (= 0 \text{ if } R(\underline{\theta}) > 0), \tag{5.80}$$

$$\iota\left(\widehat{\theta}\right) \geq 0\ \ \left(= 0 \text{ if } R\left(\widehat{\theta}\right) > 0\right), \tag{5.81}$$

$$\iota\left(\theta_j^-\right) - \iota\left(\theta_j^+\right) = \pi\left(\theta_j^+\right) - \pi\left(\theta_j^-\right) \geq (= 0 \text{ if } R(\theta_j) > 0). \tag{5.82}$$

η is continuous (see Eq. (75), p. 375, in S-S). We check that $\gamma > 0$. In addition, by continuity of η and (5.79),

$$\eta\left(\widehat{\theta}^-\right)\zeta_A\left(\widehat{\theta}\right) = \eta\left(\widehat{\theta}\right)\zeta_A\left(\widehat{\theta}\right) = 0. \tag{5.83}$$

Step 2 : We now turn to Subproblem 2. By Leibnitz's rule,

$$\partial \mathcal{W}_{A,\rho}^{C}\left(\widehat{\theta}\right)/\partial\widehat{\theta}=\frac{\partial}{\partial\widehat{\theta}}\left[\int_{\underline{\theta}}^{\widehat{\theta}}\phi_{\rho}\left(V_{A}\right)dF\left(\theta\right)\right]-\phi_{\rho}\left(V_{B}\left(\widehat{\theta}\right)-c\left(\widehat{\theta}\right)\right)f\left(\widehat{\theta}\right), \tag{5.84}$$

$$\partial \mathcal{W}_{A,\rho}^{R}\left(\widehat{\theta}\right)/\partial\widehat{\theta}=\frac{1}{F\left(\widehat{\theta}\right)}\left[\frac{\partial}{\partial\widehat{\theta}}\int_{\underline{\theta}}^{\widehat{\theta}}\phi_{\rho}\left(V_{A}\right)dF\left(\theta\right)\right]-\frac{f\left(\widehat{\theta}\right)}{F\left(\widehat{\theta}\right)}\mathcal{W}_{A,\rho}^{R}\left(\widehat{\theta}\right). \tag{5.85}$$

Eq. (79), p. 376, in (S-S) gives the value of the square brackets on the RHSs of (5.84) and (5.85) :

$$H\left(\widehat{\theta}^{-}\right)+\left[\pi\left(\widehat{\theta}\right)-\pi\left(\widehat{\theta}^{-}\right)\right]R'\left(\widehat{\theta}\right). \tag{5.86}$$

Using the continuity of x_A, z_A, f, V_A, (5.83), (5.82), $T=z_A-x_A$, and the fact that (5.6) is active at $\widehat{\theta}$, (5.84) and (5.85) reduce to (5.41) and (5.42) respectively. ■

Proof of Proposition 5.8. The results of Proposition 5.7 are also valid for $\widehat{\theta}=\overline{\theta}$ provided $R\left(\widehat{\theta}\right)=0$. We compute $\partial \mathcal{W}_{A,\rho}^{i}\left(\widehat{\theta}\right)/\partial\widehat{\theta}\Big|_{\widehat{\theta}=\overline{\theta}}<0$ and note that $\mathcal{W}_{A,\rho}^{i}\left(\overline{\theta}\right)=W_{A,\rho}^{N}$, $i=\{C,R\}$. ■

5.7.4. Simulations

We use *sufficient* conditions to construct optimal tax schedule. These conditions are equivalent to the necessary ones, provided concavity restrictions are added. By S-S (Theorem 1, p. 317-318), they read :

$$(5.56)-(5.61), \tag{5.87}$$

$$z_A'\geq 0, \tag{5.88}$$

$$H^N \text{ concave in } V_A,\ R \text{ quasi-concave in } V_A, \tag{5.89}$$

where H^N is defined by (5.55).

Our strategy is to look for candidate schedules for which ι is continuous and $z_A'\geq 0$, without taking (5.88) explicitly into account. By (5.62) and (5.63), these candidates are such that :

$$\frac{T'}{1-T'}=\left(1+\frac{1}{e}\right)\frac{1}{\theta f}\left(1-F\left(\theta\right)-\int_{\theta}^{\overline{\theta}}\pi'\left(\tau\right)d\tau-\iota\left(\overline{\theta}\right)\right)\text{ and }\pi'=f-\iota'. \tag{5.90}$$

If they satisfy all other sufficient conditions, they are then optimal. We start by noting that conditions (5.89) always hold. Indeed, quasi-concavity of R is direct. H^N is concave in V_A as $\partial^2 H^N/\partial V_A^2=0$.

We now turn to the computational procedure. We make the guess that θ^* is such that $\pi'>0$ for all $\theta>\theta^*$ at the optimal solution and look therefore for candidates having this property.

We start by choosing a value for θ^*. We use $V_A=z_A-T\left(z_A\right)-v\left(z_A/\theta\right)$, (FOIC) and (5.6) to derive z_A and $T\left(z_A\right)$ for $\theta\geq\theta^*$. We compute $T'\left(z_A\right)=\left(dT'/d\theta\right)/z_A'\left(\theta\right)$ for $\theta\geq\theta^*$. By (5.90),

$\iota\left(\overline{\theta}\right)$ and $\int_{\theta}^{\overline{\theta}}\pi'(\tau)\,d\tau$ are equal to

$$\iota\left(\overline{\theta}\right)=-\frac{T'\left(z_A\left(\overline{\theta}\right)\right)}{1-T'\left(z_A\left(\overline{\theta}\right)\right)}\frac{e}{1+e}\overline{\theta}f\left(\overline{\theta}\right), \tag{5.91}$$

$$\int_{\theta^*}^{\overline{\theta}}\pi'(\tau)\,d\tau=1-F(\theta^*)-\frac{e}{1+e}\theta^* f(\theta^*)\frac{T'(z_A(\theta^*))}{1-T'(z_A(\theta^*))}-\iota\left(\overline{\theta}\right). \tag{5.92}$$

For $\theta^*\leq\theta<\overline{\theta}$, we compute ι, derive ι' and get $\pi'=f-\iota'$. For $\theta<\theta^*$, T' are obtained from (5.90). We then compute ℓ_A. Since

$$V_A(\theta)=V_A(\underline{\theta})+\int_{\underline{\theta}}^{\theta}\ell_A^{1+1/e}(\tau)/\tau d\tau \tag{5.93}$$

by integration of (FOIC) and $T(\theta\ell_A)=\theta\ell_A-v(\ell_A)-V_A$, we have :

$$\int_{\underline{\theta}}^{\overline{\theta}}T(z_A)\,fd\theta=\int_{\underline{\theta}}^{\overline{\theta}}\left[\theta\ell_A-v(\ell_A)-V_A(\underline{\theta})-\int_{\underline{\theta}}^{\theta}\frac{\ell_A^{1+1/e}(\tau)}{\tau}d\tau\right]fd\theta,$$

which leads to

$$V_A(\underline{\theta})=\int_{\underline{\theta}}^{\overline{\theta}}\left[\theta\ell_A f-(1-F(\theta))\frac{\ell_A^{1+1/e}}{\theta}-\frac{\ell_A^{1+1/e}}{1+1/e}f\right]d\theta \tag{5.94}$$

by (TR) and Fubini's theorem. $V_A(\theta)$ is then obtained from (5.93)–(5.94). We check that $V_A>V_B-c$ for $\theta<\theta^*$, $V_A(\theta)=V_B(\theta)-c(\theta)$ for $\theta\geq\theta^*$, and $z_A'\geq 0$. If it is the case, the candidate schedule is an optimal one.

THREAT BY 0% OF THE POPULATION (BENCHMARK), $W_A = 17\ 132€$

$F(\theta)$	V_A	z_A	$T(z_A)$	$T'(z_A)$	$T(z_A)/z_A$
0.05	17 156€	3 979€	−13 203€	96.0%	−331.8%
0.50	18 028€	14 037€	− 4 487€	78.8%	− 32.0%
0.95	27 917€	57 555€	+27 917€	73.4%	+ 48.5%

THREAT BY 10% OF THE POPULATION

$W_A^N = 14\ 682€$ ($Loss = -14.3\%$); $\widehat{z}_A = 41\ 955€$; $z_A^* = 50\ 746€$; $1 - F(\theta_{\widehat{z}_A}) = 12.6\%$.

$F(\theta)$	V_A	$\Delta V_A\%$	z_A	$T(z_A)$	$T'(z_A)$	$T(z_A)/z_A$
0.05	14 709€	−14.3%	4 064€	−10 674€	95.7%	−262.6%
0.50	15 738€	−12.7%	14 526€	− 1 820€	74.9%	− 12.5%
0.95	46 970€	+73.4%	75 015€	+15 572€	00.0%	+ 20.7%

THREAT BY 5% OF THE POPULATION

$W_A^N = 15\ 622€$ ($Loss = -\ 8.8\%$); $\widehat{z}_A = 57\ 238€$; $z_A^* = 74\ 952€$; $1 - F(\theta_{\widehat{z}_A}) = 6.7\%$.

$F(\theta)$	V_A	ΔV_A	z_A	$T(z_A)$	$T'(z_A)$	$T(z_A)/z_A$
0.05	15 622€	− 8.8%	4 020€	−11 654€	96.0%	−289.9%
0.50	16 592€	− 8.0%	14 270€	− 2 869€	77.0%	− 20.1%
0.95	34 613€	+27.8%	75 015€	+27 900€	00.0%	+ 37.2%

THREAT BY 3% OF THE POPULATION

$W_A^N = 16\ 131€$ ($Loss = -\ 5.8\%$); $\widehat{z}_A = 70\ 834€$; $z_A^* = 98\ 779€$; $1 - F(\theta_{\widehat{z}_A}) = 4.4\%$.

$F(\theta)$	V_A	ΔV_A	z_A	$T(z_A)$	$T'(z_A)$	$T(z_A)/z_A$
0.05	16 156€	− 5.8	4 003€	−12 179€	96.0%	−304.2%
0.50	17 071€	− 5.3	14 174€	− 3 422€	77.8%	− 24.1%
0.95	29 570€	+ 9.2	64 652€	+29 957€	52.5%	+ 46.3%

THREAT BY 1% OF THE POPULATION

$W_A^N = 16\ 797€$ ($Loss = -\ 2.0\%$); $\widehat{z}_A = 104\ 989€$; $z_A^* = 167\ 006€$; $1 - F(\theta_{\widehat{z}_A}) = 2.0\%$.

$F(\theta)$	V_A	ΔV_A	z_A	$T(z_A)$	$T'(z_A)$	$T(z_A)/z_A$
0.05	16 821€	− 2.0	3 987€	−12 860€	96.1%	−322.6%
0.50	17 707€	− 1.8	14 082€	− 4 130€	78.5%	− 29.3%
0.95	27 636€	+ 1.0	59 413€	+28 691€	68.8%	+ 48.3%

THREAT BY 0.5% OF THE POPULATION

$W_A^N = 16\ 993€$ ($Loss = -\ 0.8\%$); $\widehat{z}_A = 126\ 638€$; $z_A^* = 261\ 228€$; $1 - F(\theta_{\widehat{z}_A}) = 1.3\%$.

$F(\theta)$	V_A	ΔV_A	z_A	$T(z_A)$	$T'(z_A)$	$T(z_A)/z_A$
0.05	17 017€	− 0.8	3 983€	−13 059€	96.1%	−322.0%
0.50	17 896€	− 0.7	14 059€	− 4 337€	78.7%	− 30.8%
0.95	27 366€	+ 1.0	59 439€	+28 279€	71.3%	+ 48.4%

THREAT BY 0.1% OF THE POPULATION

$W_A^N = 17\ 122€$ ($Loss = -\ 0.1\%$); $\widehat{z}_A = 160\ 829€$; $z_A^* = 302\ 894€$; $1 - F(\theta_{\widehat{z}_A}) = 0.7\%$.

$F(\theta)$	V_A	ΔV_A	z_A	$T(z_A)$	$T'(z_A)$	$T(z_A)/z_A$
0.05	17 146€	− 0.1	3 980€	−13 002€	96.2%	−331.5%
0.50	18 019€	− 0.1	14 041€	− 4 474€	78.8%	− 31.9%
0.95	27 158€	+ 0.3	57 726€	+27 972€	73.0%	+ 48.5%

Note : "Loss" in social welfare w.r.t. benchmark ; $\widehat{z}_A := z_A$ such that $T(z_A)/z_A$ maximum ; $z_A^* := \min z_A$ with (PC) active ; $(\widehat{z}_A, z_A^*)$ =range of decrease in $T(z_A)/z_A$ before (PC) active ; $1 - F(\theta_{\widehat{z}_A}) = \%$ agents with $T(z_A)/z_A$ decreasing ; ΔV_A :=change in V_A w.r.t. benchmark.

TAB. 5.1 – Optimum Allocations (Maximin, e=0.2, constant migration costs)

CHAPITRE 6

CONCLUSION GÉNÉRALE

La théorie de l'imposition optimale développée à la suite de Mirrlees (1971) offre un cadre d'analyse précieux afin d'étudier le dilemme entre efficacité et équité. Elle s'appuie sur le partage fondamental entre informations publiques et privées, et examine sous quelles conditions les agents ont intérêt à révéler les informations pertinentes que le décideur public ne peut directement observer.

Du point de vue théorique, les chapitres 1 et 2 utilisent la structure que la satisfaction des conditions de révélation des types place sur l'optimum social. En économie fermée et avec une population discrète, l'ensemble des contraintes d'incitation sont vérifiées du moment que les contraintes d'incitation locales vers le bas le sont. Ceci nous a permis de dériver, dans le chapitre 1, une forme réduite du problème d'imposition optimale avec préférences quasi-linéaires en consommation, dans laquelle ne figurent que les niveaux de revenu brut. La séparation claire entre efficacité et considérations éthiques au niveau des conditions d'optimalité autorise une construction géométrique simple de l'allocation optimale, en l'absence de bunching, et une analyse des propriétés de statique comparative de la solution. Les implications des conditions d'incitation sont également aux fondements de la méthodes de construction des transferts ELIE dans un environnement de second rang.

Les chapitres 4 et 5 étudient une économie ouverte dont les agents productifs peuvent s'expatrier à des fins fiscales. Ils ajoutent une contrainte supplémentaire, de participation, au problème d'imposition optimale. Dans le chapitre 4, la restriction de l'impôt à la linéarité implique l'absence de prise en compte explicite des conditions d'incitation lorsque la condition de Spence-Mirrlees est satisfaite. Dès lors, les résultats obtenus s'expliquent essentiellement par le manque de flexibilité qu'offre un impôt composé d'un revenu minimum et d'un taux marginal unique. Ce manque de degré de liberté justifie la formulation du problème d'imposition optimale non-linéaire du chapitre 5. En économie fermée, l'intérêt des agents est d'imiter leurs voisins moins productifs. Tel n'est plus le cas en économie ouverte où un individu peut être incité à se faire passer pour plus productif afin que le décideur public pense que son utilité de réservation est plus élevée. L'interaction entre conditions d'incitation et de participation complexifie les comportements mimétiques des agents. Lorsque certaines conditions sont réunies, le départ des agents les plus talentueux peut augmenter le niveau de bien-être domestique.

Les différents modèles utilisés ne sont bien sûr pas dénués de limites. Nous avons été amenés à introduire des hypothèses parfois assez restrictives. Dans les chapitres 4 et 5, nous avons ainsi étudié la politique d'imposition d'un pays donné en considérant le barème fiscal à l'étranger fixé. Dans le chapitre 1, nous avons supposé une absence d'effets revenu sur l'offre de travail. De façon générale, nous avons adopté un raisonnement bien-êtriste et considéré que les agents opéraient des calculs en marges intensives plutôt qu'extensives. Le relâchement de ces simplifications s'inscrit dans notre programme de recherche actuel.

Cette thèse développe des pistes de recherche qui mériteraient d'être davantage explorées. Deux d'entre elles nous semblent particulièrement intéressantes. La première se dégage des chapitres 1 et 2 et se concentre sur la question de l'implémentation. En économie fermée, le contrat fiscal que peut proposer le décideur public dépend fondamentalement des variables qu'il peut observer. Dans certaines activités, le temps de travail est tout à la fois observable et vérifiable. Ce n'est pas le cas dans d'autres professions. Comment dès lors poser le problème d'imposition optimale, et écrire les conditions d'incitation, dans une économie composée de ces deux types d'activités ? Dans quelle mesure le barème optimal serait-il modifié ? La seconde direction de recherche prolonge les chapitres 4 et 5. Elle consiste dans un premier temps à mieux appréhender l'impact du barème étranger sur l'impôt optimal domestique, en considérant par exemple la possibilité d'émigration des agents qualifiés vers un pays où le barème est linéaire. La second étape aborderait la question des interactions stratégiques entre décideurs publics. A cette fin, il conviendrait d'introduire des simplification sur les préférences individuelles ou l'objectif social.

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