Investment Cost Channel and Monetary Transmission *

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Abstract

We show that a standard NKM model with investment cost channels has important model stability and policy implications. Our analysis suggests that in economies characterized by supply side well as demand side channels of monetary transmission, policymakers may have to resort to a much more aggressive stand against inflation to obtain locally unique equilibrium. In such an environment targeting output gap may cause model instability. We also show that the presence of investment cost channels is enough to generate an amplification to the response of business cycle fluctuations, as the natural increase of interest rates, which are now a direct part of the firm's investment cost, curb investment and production. Furthermore, it is difficult to distinguish between the no cost channel case and labour cost channel only case in terms of dynamic behavior of macroeconomic variables . This result is important as it suggests that if one does not take into account the potential investment cost channel, one may be underestimating the importance of supply side effects.

Keyword: cost channel, investment finance, Taylor rule, indeterminacy, liquidity effects

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1 Introduction

In their seminal paper, Bernanke et al. (1996) proposed a mechanism for the amplification of business cycle fluctuations through the effects of agency costs and changes in firm's net worth on investment. This contribution, which became a workhorse for those studying the importance of credit markets in the business cycle determination, implicitly assumes an additional 'financial' channel of monetary transmission. In this model interest rates directly affect the supply side of the model as entrepreneurs must borrow to finance new investment decisions. Other contributions to monetary economics also focus on the supply or cost channel of monetary policy from the perspective of labour costs, assuming firms must borrow to pay for the wage bill in advance. (See for instance, Ravenna and Walsh (2006) and Christiano et. al. (2005)).

Are supply side effects of interest rates important? There is compelling empirical evidence that cost channels matter. Barth and Ramey (2001), for instance, show that at the manufacturing industries level strong supply-side channels in the monetary transmission are present in the short to medium run. They show that following an unanticipated monetary contraction prices rise and output falls in key US manufacturing industries after controlling for both the price puzzle and the cost effects of oil shocks. They suggest that the monetary policy may well be acting as a supply side shock to important industries in the U.S. economy. Similarly Ravenna and Walsh (2006) present corroborating econometric evidence for the direct (costly) influence of monetary policy on the U.S. inflation adjustment equation. Furthermore, Mayer and Sussman (2004) report empirical evidence that US firms rely on debt relative to equity in financing investment implying the presence of investment cost channel of monetary transmission.

Cost channels, however, are not generally present in standard New Keynesian models, where the main focus remains on interest rate channels of monetary transmission that works via intertemporal consumption decisions, often, in a cashless economy (Woodford (1999, 2003)), where credit markets are absent. In this paper we investigate in detail supply side effects of the monetary transmission, both through labour and investment, and analyze their relative importance in monetary transmission and most importantly to rational expectation equilibrium determinacy. Our particular emphasis is on the investment finance-cost channel next to the standard labour finance-cost channel of the monetary transmission mechanism. To the best of our knowledge we are the first to focus explicitly on the investment finance by providing a clear cut distinction between the impact of both labour cost finance and investment cost finance considerations. We show that the presence of investment cost channel has important model stability and policy implications.

Altering the standard New Keynesian model by incorporating supply side considerations and money-credit markets have important local determinacy implications. We first find that determinacy regions are much more narrow as compared to the literature and second the Taylor-Woodford principle is often violated. Our analysis suggests that when the monetary transmission is characterized by supply side as well as demand side channels, inflation conservatism may be paramount to obtain locally unique equilibrium, and perhaps more strikingly, output gap targeting is prejudicial, narrowing the determinacy region.[1] Furthermore, we show that the investment cost channelis the main driver of this result, hence, indeterminacy issues would be also present in financial accelerator models as suggested by Bernanke et al. (1996) and De Fiore and Tristani (2008, 2009).

One of the well known results in Bernanke et al. (1996) is that an adverse shock leads to a worsening of firm's financial condition, impairing firms access to credit, and leading to a reduction in investment and consequently production. Our simulation results suggest that the presence of investment cost channels is enough to generate an amplification to the response of business cycle fluctuations, as the natural increase of interest rates, which are now a direct part of the firm's investment cost, curb investment and production. The credit spread movements, due to financial constraints in the financial accelerator model, act on top of that. Moreover key macroeconomic variables behave in a very similar way under the full cost channel case (that is labour and investment cost channels together) and the investment cost channel case. On the other hand, it is difficult to distinguish between the no cost channel case and labour cost channel only case in terms of dynamic behavior of macroeconomic variables. This result is important as it suggests that if one does not take into account the potential investment cost channel, one may be underestimating the importance of supply side effects.

Given that our model explicitly includes a money-credit market we can also track the evolution of money holdings. Although there is ample empirical evidence on the liquidity effect of monetary policy, that is a negative comovement between money balances and nominal interest rates (Christiano et al. (1999)), this effect is difficult to obtain within standard monetary models. In these models, an exogenous expansion in money supply typically increases nominal interest rates. In practice, however, the monetary policy is conducted by interest rate announcements and open market operations such that money liquidity is determined endogenously in money markets. To obtain more realistic liquidity effects, we study movements in endogenous money balances in response to an implementation of an interest rate rule. We show that movements in money balances, that are movements in the demand for loans, are conditional on the type of exogenous shocks and on the central bank's stabilization response to these shocks. We obtain liquidity effects under aggregate supply and taste shocks. We show that policy and investment shocks fail to deliver liquidity effects. Therefore, we argue that monitoring credit market conditions are important to gauge an idea about the nature of the shocks hitting the economy.

In sum, we claim that when both supply and demand channels of monetary transmission are present, monetary policymaker's stabilization attempts may be subject to further challenges and trade-offs. We, therefore, argue that

[1] Surico (2008) reports a similar finding with a three equation New Keynesian model without investment. We show that the investment cost channel amplifies the indeterminacy problem.

monitoring the relationship between investment cost finance and money/credit conditions can potentially offer useful information for the implementation of monetary policy.

The rest of the paper is organized as follows. Section 2 presents the outline of the cash-in-advance model with labour and investment cost channel considerations. Section 3 presents the steady state of the economy. Section 4 sets out equilibrium conditions for our model economy with various cost channels. Section 5 presents the quantitative evaluation of the model where we first discuss indeterminacy conditions, then monetary policy transmission, and lastly liquidity effects. Finally, Section 6 concludes.

2 Model

The economy consists of a representative household, a firm and a financial intermediary.

2.1 Households

Formally, the household is maximizing its discounted lifetime utility given by:

$$\max_{C_t, M_{t+1}, D_t, A_t, H_t,} E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right), \quad \beta \in (0,1) \quad \sigma,\ \eta > 0 \tag{1}$$

where C_t denotes the household's total consumption, H_t denotes labour supply. The curvature parameters σ, η are strictly positive. β is the discount factor. The family faces the following budget and cash in advance constraints:

$$\begin{aligned} C_t + \frac{D_t}{P_t} + \frac{M_{t+1}^d}{P_t} + \frac{Q_{t,t+1} A_t}{P_t} &\leqslant \frac{W_t}{P_t} H_t + \frac{A_{t-1}}{P_t} + \frac{R_t D_t}{P_t} + \frac{M_t}{P_t} & (2) \\ &+ \int_0^1 \Pi_{i,t} di + \Pi_t^{FI} - T_t \\ C_t + \frac{D_t}{P_t} &\leqslant \frac{M_t}{P_t} + \frac{W_t}{P_t} H_t & (3) \end{aligned}$$

where R_t is the rate of return on the intra-period deposit D_t, M_{t+1}^d are money holdings carried over to period $t+1$, A_t represents alternative physical assets valued at the stochastic discount factor $Q_{t,t+1}$, $\int_0^1 \Pi_{i,t} di$ represents dividends accrued from the intermediate producers to households, Π_t^{FI} represents profits of the financial intermediary accrued to the household, and finally T_t stands for the lump-sum taxes households have to pay. Similar to our paper, Wang and Wen (2006) consider a case with a cash-in-advance in consumption and investment, but they do not introduce the cost channel.

The cash-in-advance constraint (CIA) imposes the condition that the household needs to allocate money balances and wage income for consumption purposes net of deposits she has decided to allocate to the financial intermediary.

The timing of deposits have important implications for the closure of the model. Here, we first assume intra- period deposits, which in effect imply the consumption Euler equation is equivalent to the one in the standard new Keynesian model. [2]

2.2 Firms

The final goods representative firm produces goods combining a continuum of intermediate goods $i \in [0,1]$ with the following production function:

$$Y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{4}$$

As standard this implies a demand function given by:

$$y_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\varepsilon} Y_t \tag{5}$$

where the aggregate price level is:

$$P_t = \left[\int_0^1 p_{i,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{6}$$

The intermediate sector is constituted of a continuum of firms $i \in [0,1]$ producing differentiated goods with the following constant returns to scale production function:

$$y_i = K_i^{\alpha} H_i^{1-\alpha} \tag{7}$$

where K is the capital stock and H is the labour used in the production. The firm hires labour and buys capital (goods) from the final good producer. It is assumed that the firm must borrow money to pay for these expenses.

To characterize the problem of intermediate firms, as standard, we split their decision into a pricing decision given the real marginal cost and then solve for their cost minimization problem.

Following the standard Calvo pricing scheme, firm i, when allowed, sets prices $P_{i,t}$ according to:

$$\max_{P_{i,t}} E_t \left\{ \sum_{s=0}^{\infty} P_{t+s} Q_{t,t+s} \omega^s y_{i,t+s} \left[\frac{P_{t,i}}{P_{t+s}} - \Lambda_{t+s} \right] \right\} \tag{8}$$

[2] An alternative specification where deposits clear next period yields an additional channel of monetary transmission through the real balance effect on consumption. In this environment consumption today is determined by the expected consumption two periods ahead $E_t(\widehat{c}_{t+2})$, and the future evolution of interest rates. This forward looking aspect when combined with the presence of cost channels has important implications for determinacy, as discussed in Aksoy, Basso, Coto Martinez (2009).

subject to the demand function (5), where Λ_t is the real marginal cost of the firm. To obtain the real marginal cost, we need to solve firm's intertemporal cost minimization problem. That is

$$\min_{K_{i,t+1},H_{i,t},} E_t \left\{ \sum_{t=0}^{\infty} Q_{t,t+1} \left({R_t}^{\gamma_1} W_t H_{i,t} + {R_t}^{\gamma_2} P_t I_{i,t} \right) \right\} \tag{9}$$

subject to the production function (7) and investment equation $I_{i,t} = K_{i,t+1} - (1-\delta)K_{i,t}$; where W_t is the nominal wage, and R_t the rate the bank charge for the loan made in period t, to be paid in $t+1$ and Λ_t is the multiplier of the constraint (7).

Expression ${R_t}^{\gamma_1} W_t H_{i,t} + {R_t}^{\gamma_2} P_t I_{i,t}$ in the cost minimization exercise characterizes the costs of firms given that they need to borrow from the financial intermediary to finance wage and investment payments[3]. Parameters $\gamma_1 \in [0,1]$, $\gamma_2 \in [0,1]$ specify the importance of the cost channel of labour and investment, respectively. Full cost channel is represented by $\gamma_1 = \gamma_2 = 1$, labour cost channel is the case with $\gamma_1 = 1$, $\gamma_2 = 0$ and if $\gamma_1 = 0$, $\gamma_2 = 1$ only the investment cost channel is present. The stochastic discount factor in period t for period t is given by $Q_{t,t} = 1$. Firms use the stochastic discount factor obtained from the household consumption condition in their production decisions.

2.3 Financial Intermediary

We assume that the financial intermediary acts in the interest of the household. That means she will optimize the discounted cash flow of the consumer. The competitive market financial intermediary (FI) gets deposits from the household and lends money to the firms in the form of loans (L). In addition to that the central bank might inject money (V) into the economy by giving it directly to the financial intermediary, who will then add to the other funds used for lending. At the end of each period the money injected then forms part of the household assets (as money holdings). Formally the financial intermediation profits, which are part of the household budget constraint (given V_t), are given by:

$$\underset{D_t}{Max} R^L_{1,t} D_t - R_{1,t} D_t$$

$$R^L_{1,t} = R_{1,t}.$$

Loan market equilibrium requires:

$$L^s(V_t, R^L_{1,t}) = L^D(R^L_{1,t}).$$

[3] Note that as $R^{\gamma_1}WH \approx WH + (R-1)\gamma_1 WH$ the cost function used in the firm's problem is equivalent to having the firm paying the full labour costs and the net interest rate on the portion $\gamma_1 WH$ that needed to be borrowed. That implies the firm has only a portion $(1-\gamma_1)$ of the labour costs at its disposal at the beginning of the period, when wages must be paid. The same applies for investment.

In equilibrium the demand for credit to pay the production input must be equal to the supply of credit made by the banking system. The credit supply is determined by deposits and the monetary injection. Therefore, the credit market condition is given by:

$$\gamma_1 W_t H_t + \gamma_2 P_t I_t = D_t + V_t. \tag{10}$$

Once again, $\gamma_1 > 0$ implies firms must borrow to pay the wage bill and $\gamma_2 > 0$ implies firms must borrow to invest.

The money supply is determined by the government. Then, the government budget constraint is given by:

$$T_t + V_t = 0, \tag{11}$$

where T stands for taxes received. Note that any profit accruing to the FI due to monetary injections are transferred to the household by the end of the period in the form of financial intermediary dividends.

2.4 Equilibrium

The equilibrium of the economy is defined as the vector of Lagrange multipliers $\{\lambda_t, \mu_t, \Lambda_t\}$, the allocation set $\{C_t, H_t, K_{t+1}, D_t, M_{t+1}, Y_t\}$, and the vector of prices $\{p_{i,t}, P_t, W_t, R_t\}$ such that the household, the final good firm and intermediate firms maximization problems, the market clearing conditions and the government budget constraint hold.

Consumer problem is represented by the standard Euler conditions[4]:

$$\beta E_t \left(\frac{R_t C_{t+1}^{-\sigma}}{\pi_{t+1}} \right) = C_t^{-\sigma} \tag{12}$$

$$\frac{\chi H_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \tag{13}$$

From the consumer problem we obtain the stochastic discount factor:

$$Q_{t,t+1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma} \pi_{t+1}} \right].$$

The goods market clearing condition is given by:

$$Y_t = C_t + I_t. \tag{14}$$

The capital and labour market clearing condition are given by:

$$K_t = \int_0^1 K_{i,t} di \text{ and } H_t = \int_0^1 H_{i,t} di. \tag{15}$$

[4] The Euler equation stated holds as an equality as long as $D_t > 0$, which is the case at equilibrium.

Investment evolves according to:

$$I_{i,t} = K_{i,t+1} - (1-\delta)K_{i,t}. \tag{16}$$

The price setting equation is given by solving (8):

$$P_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} P_{t+s} Q_{t,t+s} \omega^s \Lambda_{t+s} y_{i,t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} P_{t+s} Q_{t,t+s} \omega^s \frac{y_{i,t+s}}{P_{t+s}} \right\}} \tag{17}$$

Finally, from the firm problem we obtain the demand for capital and labour and the optimal price. Next, we describe the three main cases of cost channels through which nominal rates affect real marginal cost of the firm.

Full Cost Channel: After rearranging first order conditions and using the stochastic discount factor $Q_{t,t+1} = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma} \pi_{t+1}} \right]$, we obtain the following equilibrium conditions under the full cost channel:

$$\Lambda_t = \frac{R_t^{\gamma_1} W_t H_{i,t}}{P_t Y_{i,t}(1-\alpha)} \tag{18}$$

$$R_t^{\gamma_2} = \beta E_t \left\{ \frac{R_t}{\pi_{t+1}} \left[\Lambda_{t+1} \frac{\alpha Y_{i,t+1}}{K_{i,t+1}} + (1-\delta) R_{t+1}^{\gamma_2} \right] \right\}. \tag{19}$$

As conditions (18) and (19) reveal, when both cost channels of labour and investment are present, the real marginal cost of the firm will be, among others, a function of both current and future expected short term rates. The investment cost channel also reveals the impact of the expected labour supply decisions on the real marginal cost.

Cost Channel in Labour: Ravenna and Walsh (2006) derive their aggregate supply equation based on the impact of policy changes on labour cost financing leaving aside the impact of the policy decisions on the investment finance costs. To get Ravenna and Walsh (2006), we need to remove the cost channel in investment, i.e. $\gamma_2 = 0$. Setting $\gamma_2 = 0$ we obtain:

$$\Lambda_t = \frac{R_t^{\gamma_1} W_t H_{i,t}}{P_t Y_{i,t}(1-\alpha)} \tag{20}$$

$$1 = \beta E_t \left\{ \frac{R_t}{\pi_{t+1}} \left[\Lambda_{t+1} \frac{\alpha Y_{i,t+1}}{K_{i,t+1}} + (1-\delta) \right] \right\}. \tag{21}$$

As revealed in conditions (20) and (21), in the case of the labour cost channel only, expected labour supply and nominal rates in period t still affect the real marginal cost. Carlstrom and Fuerst (2005) present a model without cash-in-advance and with capital accumulation. Note that their equilibrium conditions are recursive, while in our model cash-in-advance constraint leads to an impact of the expected two period ahead inflation rate on real variables.

Cost Channel in Investment: By assuming $\gamma_1 = 0$ we obtain the case for the cost channel only in investment (see Kurozami and Van Zandweghe (2008)):

$$\Lambda_t = \frac{W_t H_{i,t}}{P_t Y_{i,t}(1-\alpha)} \quad (22)$$

$$R_t^{\gamma_2} = \beta E_t \left\{ \frac{R_t}{\pi_{t+1}} \left[\Lambda_{t+1} \frac{\alpha Y_{i,t+1}}{K_{i,t+1}} + (1-\delta) R_{t+1}^{\gamma_2} \right] \right\}. \quad (23)$$

In the case of the investment cost channel only, both current and expected rates influence real marginal costs. Note that Dow (1995) obtains a similar expression for investment using a slightly different discount factor, since firms have to pay the capital input in advance and an increase in nominal interest rates raises the capital cost.

To establish the money market condition we need an expression that links short term rates and monetary injections (V_t). We obtain this by combining the cash-in-advance specification (3) and the credit market clearing condition (10). At the credit market equilibrium, the demand for credit to pay the production input must be equal to the supply of credit made by the banking system. The credit supply is determined by the deposits and the monetary injection. Therefore, the credit market condition is given by:

$$\gamma_1 W_t H_t + \gamma_2 P_t I_t = D_t + V_t. \quad (24)$$

Combining this result with the CIA constraint and money market flow equation gives the following money/credit condition:

$$(\gamma_1 - 1) \frac{W_t H_t}{P_t} + \gamma_2 I_t + C_t = \frac{M_{t+1}}{P_t}. \quad (25)$$

3 Steady State with Flexible Prices and Zero Inflation

The New Keynesian literature considers that the government implements zero inflation at the steady state (see Ravenna and Walsh (2006)). While we retain this assumption, it is important to note that even in the steady-state with flexible prices, the monetary policy is non-neutral. The reason is that the cost channel implies a steady state relation between investment/employment and the nominal rate, which in turn is determined by the inflation rate. To show this we write from the consumption Euler condition:

$$R = \frac{\pi}{\beta} \quad (26)$$

where R and π are directly associated. We are going to normalize the aggregate price level $P = 1$. When wage payments are made at beginning of the period we obtain the conventional labour supply condition:

$$\frac{\chi H^{\eta}}{C^{-\sigma}} = \omega$$

where ω is now the real wage.

Finally in the case of full cost channel we show the effect of inflation in the firm employment and investment decision. At the symmetric equilibrium, price over marginal cost is given by $P = \frac{1}{\mu} MC$, since $P = P_i$. Now, since Λ is the real marginal cost and $\mu = \frac{MC}{P} = \Lambda$, at the steady state marginal cost relations read as:

$$\mu = \omega \frac{H}{Y(1-\alpha)} \frac{\pi}{\beta} \tag{27}$$

$$\delta \frac{\pi}{\beta} = \mu \frac{\alpha Y}{K} \tag{28}$$

As we see from (27 and (28) the policymaker actually could inflate away mark-up distortion in the economy. In what follows we ignore price inflation effects on the real economy under flexible prices, i.e. we will work with log-linearised model around a zero inflation steady state (i.e., $\pi = 1$) as in Ravenna and Walsh (2006).

4 The Linearized Model

The linear model, based on equilibrium conditions (7), (12), (14) - (19) and (25) , for the set of variables $\left\{ \widehat{c}_t, \widehat{r}_t, \widehat{\Lambda}_t, \widehat{y}_t, \widehat{\pi}_t, \widehat{i}_t, \widehat{k}_{t+1}, \widehat{h}_t, \widehat{m}_t \right\}$ is summarized as follows:

$$\widehat{c}_t = E_t\left(\widehat{c}_{t+1}\right) - \frac{1}{\sigma}E_t\left[\widehat{r}_t - \widehat{\pi}_{t+1}\right] \tag{29a}$$

$$\gamma_2 \widehat{r}_t = E_t\left[\widehat{r}_t - \widehat{\pi}_{t+1}\right] + \tag{29b}$$

$$+(1-\beta(1-\delta))E_t\left[\widehat{y}_{t+1} + \widehat{\Lambda}_{t+1} - \widehat{k}_{t+1}\right] + \beta\gamma_2\left(1-\delta\right)E_t\left(\widehat{r}_{t+1}\right)$$

$$\widehat{\Lambda}_t = \gamma_1 \widehat{r}_t + \left(1+\eta\right)\widehat{h}_t + \sigma\widehat{c}_t - \widehat{y}_t \tag{29c}$$

$$\widehat{\pi}_t = \beta E_t\left(\widehat{\pi}_{t+1}\right) + \kappa\widehat{\Lambda}_t \tag{29d}$$

$$\widehat{y}_t = s_c \widehat{c}_t + s_I \widehat{i}_t \tag{29e}$$

$$\widehat{k}_{t+1} = (1-\delta)\widehat{k}_t + \delta\widehat{i}_t \tag{29f}$$

$$\widehat{y}_t = \alpha\widehat{k}_t + (1-\alpha)\widehat{h}_t \tag{29g}$$

$$\widehat{m}_{t+1} = \widehat{m}_t - \widehat{\pi}_t + v_t \tag{29h}$$

$$\left[\begin{array}{ccc} \left(\gamma_1 - 1\right)\left(\widehat{\Lambda}_t - \gamma_1\widehat{r}_t\right) & = & \frac{m/Y}{\varpi}\widehat{m}_{t+1} - \frac{s_c}{\varpi}\widehat{c}_t - \frac{\gamma_2 s_I}{\varpi}\widehat{i}_t - y_t \quad \text{for} \quad \gamma_1 \neq 1 \\ \widehat{m}_{t+1} & = & \frac{s_c}{\left(s_c + \gamma_2 s_I\right)}\widehat{c}_t + \frac{\gamma_2 s_I}{\left(s_c + \gamma_2 s_I\right)}\widehat{i}_t \quad \text{for } \gamma_1 = 1 \end{array}\right] \tag{29i}$$

where $\kappa = (1-\omega)(1-\omega\beta)/\omega$, $s_c = C/Y$, $s_I = I/Y$ and $\varpi = m/Y - \left(s_c + \gamma_2 s_I\right)$.

Equation (29h) shows the flow of real money balances. Under this framework monetary policy is exogenously set through monetary injections. However, as our interest is in active monetary policy, we replace this equation with an interest rate rule of the form

$$\widehat{r}_t = \epsilon_y \widehat{y}_t + \epsilon_\pi \widehat{\pi}_t + \epsilon_r \widehat{r}_{t-1} + v_t. \tag{30}$$

Note that we still keep the credit market equilibrium equation (29i) since that allows us to track the evolution of real money balances.

The natural benchmark model to compare our results with is the New Keynesian model (NKM) for a cashless economy with investment, which is effectively equal to our economy when $\gamma_1 = \gamma_2 = 0$, with no cost channel.

4.1 Parameter Values

The linearized model has 12 free parameters: $\sigma, \gamma_1, \gamma_2, \delta, \eta, s_c, s_I, \alpha, \beta, \omega, \epsilon_v$ and ρ. We set the parameter of intertemporal elasticity of substitution $\sigma = 1$ and the parameter of intertemporal elasticity of labour supply $\eta = 1.03$. The discount factor, β, is calibrated to be 0.99, which is equivalent to an annual steady state real interest rate of 4 percent. The depreciation rate, δ, is set equal to 0.05 per quarter. We set $\alpha = 0.36$ which roughly implies a steady state share of labour income in total output of 66%. The share of steady state consumption (s_c) is set equal to 0.625, while the share of steady state investment (s_I) is set equal to 0.275. ϵ_v represents the parameter for the credit-money markets. We resort to Christiano and Eichenbaum (2005) in calculating the parameter who report that the steady state velocity of money $\frac{m}{Y} = 0.44$. This implies in our case a value for $\epsilon_v = \frac{m}{m-Y} = 1.7857$. Parameters γ_1, γ_2 regulate the importance of labour and investment cost channels, where $\gamma_{1,2} = 1$ implies full-cost channel, $\gamma_{1,2} = 0$ implies no-cost channel. Throughout simulations shocks are persistent. The parameter ρ represents the persistence of shocks and we set this equal to 0.5 in all simulations. We set the value of the Calvo parameter ω (fraction of firms which do not adjust their prices) as equal to 0.66 consistent with the findings reported in Gali and Gertler (1999).

5 Model Determinacy

The introduction of investment and labour cost channels not only have implications to monetary transmission but perhaps more importantly, impact the local stability properties of the model economy. Hence, before proceeding to present details of the monetary transmission we first focus on interest rate rules and uniqueness of rational expectations equilibrium. This is necessary as in order to study monetary transmission we need to assume the appropriate interest rate rule parameters that ensure model stability.

Taylor (1993) provided theoretical arguments for why an inflation coefficient greater than one is crucial to macroeconomic stabilization. In this framework, a more than one-to-one increase in nominal rates in response to an increase in expected inflation effectively raises real interest rates; therefore a decline in aggregate demand alleviates inflationary tendencies. Woodford (2003) formally discusses conditions for determinacy of equilibrium within the setting of a cashless New Keynesian framework (Taylor-Woodford principle). He argues that when a monetary policymaker targets output gap next to inflation she effectively relaxes the conditions for equilibrium determinacy. He also shows that interest rate smoothing is useful in obtaining a locally unique equilibrium. While we concur that interest rate smoothing is indeed important to achieve a unique local equilibrium, targeting output gap is in fact counter productive for determinacy purposes. We find that uniqueness of equilibrium is harder to obtain in the presence of cost channels and money-credit markets. Both cost channels are important for this result. However, we find that the presence of investment cost channel narrows down the parameter space that the policymaker can use to stabilize the economy more significantly than the labour cost channel.

The first figure illustrates the effect of the two types of cost channel on indeterminacy when the monetary policy rule has no interest rate inertia ($\epsilon_r = 0$) and targets inflation and output ($\epsilon_y = 0.5$). While in the standard New Keynesian model the monetary authority ensures uniqueness responding more than one to one to an inflation deviation, that is not sufficient in the case both cost channels are in place; the inflation parameter must be greater than 1.6 to ensure determinacy. Although both labour and invest cost channels contribute to that result, an economy with investment cost channel requires a stronger response to inflation deviation than when only the labour cost channel is present.

[Insert Figure 1 here]

Figure 2 focus on the effect of interest rate smoothing on indeterminacy when cost channels are present. Cost channels have little effect on indeterminacy comparing to a standard New Keynesian model. In both cases, interest rate smoothing helps increasing the determinacy region (we present full and no cost channel cases only).

[Insert Figure 2 here]

Finally, in figure 3 we look at the determinacy effects of altering the monetary policy response to output deviations (ϵ_y) when cost channels are present. As figure 3 (d) shows, increasing output targeting has a mildly positive effect, increasing the determinacy region, in the benchmark New Keynesian model. On the other hand, when cost channels are present, increasing the output gap parameter decreases the region in which the equilibrium is unique. When both channels are present, and $\epsilon_y = 1$, the monetary authority can not guarantee stability even if it changes interest rates by two times the inflation deviation. Once again, although both cost channels are important for this result, the investment cost channel appear to contribute more than the labour cost channel.

We motivate this as follows: as in our model a contractionary policy change leads to a contraction of the economy and two opposing implications for inflation, decreasing through the demand channel an increasing through the supply channel. Targeting output together with inflation requires the determinacy concerned policymaker to act in order to make sure aggregate demand channels dominate the cost channels. Surico (2008) finds similar results using a restricted three equation NK model with a cost channel in wage payments.

[Insert Figure 3 here]

We conclude that indeterminacy problems are more severe within these model settings. The Taylor-Woodford principle that prescribes simple conditions for ensuring macroeconomic stability is often violated. Therefore, if the macroeconomic environment includes supply side as well as demand side considerations together with a role for money credit markets, very aggressive stand against inflation may be paramount in achieving policy outcomes that yield model determinacy.

6 Monetary Transmission

The analysis of the stability of model indicated that an interest rate rule with parameters $\epsilon_y = 0.5$, $\epsilon_\pi = 1.5$ delivers model stability under all types of cost channels if and only if there is strong interest rate smoothing. Therefore we will run model simulations where the interest rate coefficient is equal to 1 or $\epsilon_r = 1$.

Before we study the statistical properties and impulse responses of key variables with respect to a range of shocks we briefly recall our model's implications for monetary transmission. Assumptions we made imply an important role for investment and production costs in the monetary transmission. As argued in the introduction, most NK models assume that monetary transmission actually occurs through an intertemporal substitution channel in consumption. If the policymaker follows a Taylor rule, this modelling approach implies that the policymaker faces no trade off between real output and inflation under aggregate demand shocks. In contrast, here we allow for supply side implications of policy changes thereby introducing a trade off between real output and inflation. Ultimately, the outcome of policy changes will depend on the relative importance of aggregate demand versus aggregate supply channels.

In order to compare different types of cost channels we analyze the response of key macroeconomic variables given a money shock for following four cases:

- full cost channel ($\gamma_1 = \gamma_2 = 1$),
- no cost channel or New Keynesian model (NKM model) with investment ($\gamma_1 = \gamma_2 = 0$),
- only labour cost channel ($\gamma_1 = 1$ and $\gamma_2 = 0$),
- only investment cost channel ($\gamma_1 = 0$ and $\gamma_2 = 1$).

We consider four sets of independently and identically distributed shocks:

- a taste shock ($\varepsilon_{c,t}$) directly associated with the consumption Euler equation,
- an investment shock ($\varepsilon_{I,t}$) that reflects an unexpected boost in investment,
- an inflation (or supply) shock ($\varepsilon_{\pi,t}$) associated with the New Keynesian Phillips Curve
- a policy shock ($\varepsilon_{r,t}$) to the Taylor Rule.

All shock processes are assumed to have an autocorrelation coefficient equal to 0.5; their standard deviations are set equal to 1%.

In conducting simulations we assume that the economy is hit by each one of the four different types of shocks one at a time. In Table 1 we present the correlation structure between real output and key macroeconomic variables, in Table 2 absolute standard deviations of key macroeconomic variables and in Table 3 the relative standard deviations of these variables with respect to real output. We compare correlations and standard deviations statistics with the US data as reported by Stock and Watson (1999) in the final column. In reporting the data we refer to non-durables consumption as a proxy for consumption, total fixed investment as a proxy for investment, consumer price index as a proxy for inflation, federal funds rate for nominal short term rates and finally real gross domestic product as a proxy for real output.

We compare the results for four basic models:

Table 1: Contemporaneous Correlations with output (in %)

	$c-shock$	$i-shock$	$\pi-shock$	$r-shock$	*Data*
Full Cost					
c	6	-13	47	83	74
i	74	78	92	88	82
π	-98	-94	-99	91	35
r	-99	-10	-94	84	38
Investment Cost					
c	-24	-37	49	89	74
i	76	83	92	93	82
π	-98	-93	-99	97	35
r	-99	19	-94	93	38
Labour Cost					
c	-24	-32	54	34	74
i	76	80	91	95	82
π	-97	-89	-99	95	35
r	-99	29	-95	89	38
No Cost					
c	-27	-38	49	33	74
i	76	84	92	96	82
π	-97	-91	-99	95	35
r	-99	37	-95	89	38

Table 2 : Standard Deviations (in %)

	$c-shock$	$i-shock$	$\pi-shock$	$r-shock$
Full Cost				
c	4.4	3.3	2.1	0.4
i	17.4	13.6	12.3	1.1
π	1.3	0.9	1.4	0.9
r	1.3	0.7	0.8	0.5
y	3.7	2.6	3.8	0.5
Investment Cost				
c	4.1	3.1	2.1	0.4
i	16.2	13.9	12.2	1.5
π	0.9	0.8	1.3	0.8
r	0.9	0.6	0.8	0.4
y	2.7	2.3	3.8	0.7
Labour Cost				
c	3.8	3.7	2.0	0.3
i	14.9	15.5	11.1	2.4
π	0.9	0.8	1.3	0.8
r	0.9	0.8	0.7	0.5
y	2.5	2.6	3.6	0.7
No Cost				
c	3.6	3.6	1.8	0.3
i	14.0	16.0	10.6	2.4
π	0.8	0.9	1.2	0.8
r	0.8	0.7	0.7	0.5
y	2.2	2.7	3.3	0.7

Table 3 : Relative Standard Deviations vis-a-vis output $\left(\frac{\sigma_x}{\sigma_y}\text{ in }\%\right)$

	$c-shock$	$i-shock$	$\pi-shock$	$r-shock$	$Data$
Full Cost					
c	117	127	55	75	67
i	462	526	321	228	299
π	35	34	35	174	87
r	34	28	21	95	89
Investment Cost					
c	155	134	56	66	67
i	607	592	318	223	299
π	35	33	35	123	87
r	34	27	21	67	89
Labour Cost					
c	153	145	56	41	67
i	605	605	309	342	299
π	36	33	36	114	87
r	36	31	21	75	89
No Cost					
c	163	132	55	40	67
i	634	592	318	343	299
π	36	33	36	112	87
r	37	27	20	74	89

In Table 1 we report contemporaneous correlations of key macroeconomic variables with real output. It is evident that the nature of the shocks hitting the economy is of paramount importance in assessing the comovements. Several observations are due:

First, in the case of a taste shock, only the full cost channel model is able to deliver a positive association between output and consumption. In all other models, including the New Keynesian model (no cost channel model) consumption and investment are negatively correlated. This result also obtains when the economy is hit by investment shocks. Here, however, even the full cost channel model delivers a negative comovement between investment and consumption. In other words, if the economy is subject to taste and investment shocks, consumption appears to be a countercyclical variable, while investment appears to be the clearly dominant driving variable for output fluctuations. Second, to recover the positive association between consumption and output as found in the data, aggregate supply shocks and policy shocks are necessary. Here inclusion of investment cost channel becomes even more important as the US data shows a very strong correlation (74%) between consumption and output. Our simulation suggest that this is only possible if the economy is subject to policy shocks with investment cost channel. In the case of labour cost channel only or no cost channels at all , even a policy shock fails to deliver meaningful correlation patterns between consumption and output.

In Table 2 we report absolute standard deviations for key variables of concern. We first note that taste shocks and inflation shocks yield higher volatility across all variables when the economy is subject to investment or both cost channels. Inflation becomes more volatile in the presence of investment cost channels when the policymaker is attempting to stabilize output as well as inflation.

Investment and policy shocks yield lower volatilities in real output and its main driving component, investment, in the case of full or investment cost channels as opposed to labour cost and no cost channels. We argue that this is because investment cost channel acts like an investment adjustment cost dampening its response to investment shocks. Recent empirical microeconomic evidence rejects the existence of capital or investment adjustment costs of the format usually employed in the macroeconomic literature (Groth and Khan (2007)). Therefore, our results suggest that one avenue of research is to develop cash flow or credit constraints to investments that can generate the observed responses to the main macroeconomic variables as the currently employed adjustment costs do but are more realistic at the microeconomic level.

In Table 3 we report relative standard deviations of key macroeconomic variables vis-a-vis real output. All models mimic typical rankings of relative volatilities of key variables found in actual data. In other words, we find that investment is more volatile than real output and real output is more volatile than consumption in all models. However, exclusion of investment cost channels imply much lower relative standard deviations of consumption vis-a-vis real output in the case of policy shock. We suggest that to improve the fit with the data we need an explicit investment cost channel.

To complete the analysis we present impulse response analysis of all four models that are subject to four different shocks as outlined above.

[Insert Figure 4 here]

Figure 4 shows the comparison of consumption, investment, output and inflation with respect to a monetary policy shock. Here, the output-inflation trade off result is absent. In other words, a 1% std. monetary policy shock (an increase) yields a decline in both inflation and output in line with standard New Keynesian arguments. The impulse responses of the model with full-cost channel and the investment cost channel only are similar to each other but quite different to the responses observed for the models with no cost channel and labour cost channel, which are themselves quite similar.

As Christiano et. al. (2005) point out, one of the main discrepancies of the New Keynesian model with investment in relation to the data is that after a contractionary monetary policy shock, investment moves to strongly driving output down but consumption responses are quite flat. In order to correct this anomaly they introduce investment adjustment costs. As Figure 4 clearly shows the models without investment cost channel are subject to the same problem. This is not the case when the investment cost channel is present, investment and output respond less and consumption also falls after a monetary policy shock[5].

[Insert Figure 5 here]

[5]Note, however, that the model with investment adjustment cost generates hump-shaped investment responses, while the model with the investment cost channel in this simple form does not.

Figure 5 shows the responses to a taste shock. A positive shock leads to crowding out of investment, i.e. it leads to an increase in consumption and a decrease in investment in all cases. As the investment dominates the consumption effect, output declines while inflation goes up. This is because when initially consumption goes up inflationary pressures build up. To counter these interest rates tend to go up depressing investment and therefore output. As interest rate affects inflation directly in the case of cost channels the policymaker confronts the output-inflation trade-off, since, by the definition of the Taylor rule, the policymaker takes into account both inflation and real output deviations from target. While all key macroeconomic variables responses are qualitatively similar under different assumptions about the cost channel, it appears that inflation and output responses are largest under full cost channel case followed by case with the investment cost channel only.

[**Insert Figure 6 here**]

Figure 6 shows impulse responses under an investment shock. In this case, again there is crowding out. This time, however, consumption decreases as investment increases. Similar to a taste shock, an investment shock triggers a trade off between inflation and output. The full cost channel case delivers smallest deviations from the target for both inflation and output, followed by the investment cost channel case. Interestingly, no cost channel case (NKM model) yields largest 'loss' in terms of output and inflation. Thus, the presence of cost channels dampen the impact of demand shocks.

[**Insert Figure 7 here**]

Figure 7 shows impulse responses under an inflation shock. An aggregate supply shock, i.e. a shock on inflation, yields a very similar qualitative picture as in the case of a taste shock. Again key macroeconomic variables move very similarly in the case of full cost channel and investment cost channel. Aggregate supply shocks imply output inflation trade-off. In this case inflation and output responses are largest under full cost channel case followed by case with the investment cost channel only. Thus, the presence of cost channels amplify the impact of supply shocks.

In sum, our simulation results suggest that key macroeconomic variables behave in a very similar way under the full cost channel case and investment cost channel cases. Similarly, it is difficult to distinguish between the no cost channel case and labour cost channel only case in terms of dynamic behavior of macroeconomic variables . This result is important as it suggests that if one does not take into account the potential investment cost channel, one may be underestimating the importance of cost channels.

7 Liquidity Effects

In this section we discuss liquidity effect results with an active policymaking. In other words, liquidity conditions are determined endogenously where the policymaker implements Taylor Rules as given by Equation (30). In figure 8 we present the liquidity responses (money balances) of the economy together with the associated nominal interest rates under a series of shocks on consumption (taste), investment, inflation and interest rates. We report only cases with full cost channels, as the results with the inclusion of cost channels separately do not influence the qualitative findings. $Money$ stands for the nominal money balances and $Interest\ Rate$ for nominal interest rates.

[Insert Figure 8 here]

We observe that after a taste shock (a shock in consumption) nominal rates rise. This is consistent with the observation that as consumption increases, credit demand (liquidity) and inflationary pressures on the side of the central bank increase. To curb this effect the central bank needs to raise interest rates. This amounts, together with an increase in real wages, to a decline in investment as current and expected future marginal costs increase (see (19)). The net effect on output is a contraction. In other words, if liquidity effect represents the credit conditions in the economy, a taste shock causes a contractionary monetary policy and a strong decline in the credit demand; equivalent to a liquidity contraction as one would expect with liquidity effects. In the case of investment shocks, however the effect on consumption and investment are now reversed, while the net output (therefore credit demand) increases leading to an increase in output leading to inflationary pressures; as a consequence interest rates go up. Therefore, we do not observe the liquidity effect.

In the case of an inflation shock, i.e. a shock to the Phillips Curve, we observe an immediate increase in the nominal interest rates due to stabilization preferences of the central bank. A contractionary monetary policy leads to an increase in consumption while investment start to decrease. Consumption increases due to intertemporal elasticity concerns (future value of consumption will be relatively eroded), while the investment demand goes down as both expected inflation and a rise in the policy interest rate cause the current and future expected marginal cost to increase. The sharp decline in investment credit demand is not met by an increase in consumption demand. In turn, the rise in the policy rates due to supply shocks is associated with a sharp decline in the demand for credit to finance investment and labour costs. Therefore, the liquidity effect obtains. Here again, as in the case of a taste shock, the liquidity effects of the shock is amplified in the presence of investment cost channels. Therefore, we argue that it is important to make a distinction between investment and labour cost channels if we are interested in the size of liquidity responses.

In the case of policy shocks, i.e. a shock to the Taylor Rule, we do not observe the liquidity effect. In fact, we observe that nominal rates initially respond negatively to the shock in the Taylor Rule. While, at first, this may seem puzzling, we argue that this is exactly what should happen in equilibrium. First, note that we are initially at the steady state where both inflation and output gap are set equal to zero. A positive shock in interest rates means that the current output and inflation both will decrease below their steady state values, or below what the central bank prefers these to be. From the expression of the marginal cost and associated investment equation we observe that investment should go down as the expected real marginal cost will increase in the economy while consumption should likewise decrease due to intertemporal substitution effects. The forward looking nature of both investment and consumption pins down the aggregate goods and credit demand in our model. The only way to avoid output and inflation to fall below their potential values is to cut the policy rates. That is exactly what happens in the model. Finally, similar to taste and inflation shocks, in the case of a policy shock the presence of investment cost channel amplifies the liquidity response.

We conclude this section by arguing that monitoring credit market conditions are important to gauge an idea about the nature of the shocks hitting the economy.

8 Conclusions

In this paper we present a stylized model with investment cost channels. We show that altering an otherwise standard NKM model for investment cost channels has important model stability and policy implications.

We first find that the model indeterminacy becomes a major issue when supply side considerations are taken seriously. Our analysis suggests that in economies characterized by supply side well as demand side channels of monetary transmission, policymakers may have to resort to a much more aggressive stand against inflation to obtain locally unique equilibrium. In such an environment targeting output gap may cause model instability. Furthermore, we show that the investment cost channel, also assumed by Bernanke et al. (1996), is the main driver of this result, hence, indeterminacy issues would be also present in financial accelerator models.

Secondly, we show that investment cost channels deliver a business cycle amplification of small shocks. One of the well known results in Bernanke et al. (1996) is that an adverse shock leads to a worsening of firm's financial condition, impairing firms access to credit, and leading to a reduction in investment and consequently production. Our simulation results suggest that the presence of investment cost channels is enough to generate an amplification to the response of business cycle fluctuations, as the natural increase of interest rates, which are now a direct part of the firm's investment cost, curb investment and production. The credit spread movements, due to financial constraints in the financial accelerator model, act on top of that. Moreover, key macroeconomic variables behave in a very similar way under the full cost channel case (that is labour and investment cost channels together) and the investment cost channel case. On the other hand, it is difficult to distinguish between the no cost channel case and labour cost channel only case in terms of dynamic behavior of macroeconomic variables. This result is important as it suggests that if one does not take into account the potential investment cost channel, one may be underestimating the importance of supply side effects.

Finally, we look at that liquidity effects in the model. Given that our model explicitly includes a money-credit market we can also track the evolution of money holdings. Although there is ample empirical evidence on the liquidity effect of monetary policy, this effect is difficult to obtain within standard monetary models. We obtain liquidity effects under aggregate supply and taste shocks. We show that policy and investment shocks fail to deliver liquidity effects. Therefore, we argue that monitoring credit market conditions are important to gauge an idea about the nature of the shocks hitting the economy.

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9 Appendix: Log-linear Approximations:

Remember Campbell calculus. Suppose we have:

$$y_t = f(x_t, y_t)$$

Then, we should have that:

$$\overline{y}\widehat{y}_t = f_x(\overline{x}, \overline{z})\overline{x}\widehat{x}_t + f_z(\overline{x}, \overline{z})\overline{z}\widehat{z}_t$$

which is often written as:

$$\begin{aligned} \widehat{y}_t &= \frac{f_x(\overline{x}, \overline{z})\overline{x}}{\overline{y}}\widehat{x}_t + \frac{f_z(\overline{x}, \overline{z})\overline{z}}{\overline{y}}\widehat{z}_t \\ &\text{or} \\ \widehat{y}_t &= \frac{f_x(\overline{x}, \overline{z})\overline{x}}{f(\overline{x}, \overline{z})}\widehat{x}_t + \frac{f_z(\overline{x}, \overline{z})\overline{z}}{f(\overline{x}, \overline{z})}\widehat{z}_t \end{aligned}$$

1. Log-linearisation of the consumption Euler condition (12):

$$E_t\left(\widehat{c}_t\right) = E_t\left(\widehat{c}_{t+1}\right) - \frac{1}{\sigma} E_t\left[\widehat{r}_t - \widehat{\pi}_{t+1}\right] \tag{31}$$

2. Labour supply equation (13):

$$\eta \widehat{h}_t + \sigma \widehat{c}_t = \widehat{w}_t - \widehat{p}_t \tag{32}$$

3. Log-linearization of the investment equation (19). Note that $\beta E_t \frac{(C_{t+1}^{-\sigma})}{(C_t^{-\sigma})\pi_{t+1}} = Q_{t,t+1} = \frac{1}{R_t}$. Log-linearization of this gives us $\widehat{q}_{t,t+1}$

$$\gamma_2 \widehat{r}_t \approx \widehat{q}_{t,t+1} + E_t\left(\widehat{\pi}_{t+1}\right) + \ln E_t\left[\Lambda_{t+1}\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)R_{t+1}^{\gamma_2}\right]$$

Define $F_t = \ln E_t\left[\Lambda_{t+1}\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)R_{t+1}^{\gamma_2}|\Omega_{t,1}\right]$ and at the steady state $F = \left[\Lambda\frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2}\right]$

$$\widehat{F}_t = \frac{f_y(\overline{x},\overline{z})\overline{y}}{f(\overline{x},\overline{z})}\widehat{y}_{t+1} + \frac{f_\Lambda(\overline{x},\overline{z})\overline{\Lambda}}{f(\overline{x},\overline{z})}\widehat{\Lambda}_{t+1} + \frac{f_r(\overline{x},\overline{z})\overline{r}}{f(\overline{x},\overline{z})}\widehat{r}_{t+1} + + \frac{f_k(\overline{x},\overline{z})\overline{k}}{f(\overline{x},\overline{z})}\widehat{k}_{t+1} \tag{33}$$

$$\begin{aligned}\widehat{F}_t &= \frac{\Lambda\frac{\alpha}{K}Y}{\left[\Lambda\frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2}\right]} E_t\left[\widehat{y}_{t+1} - \widehat{k}_{t+1}\right] + \frac{\frac{\alpha Y}{K}\Lambda}{\left[\Lambda\frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2}\right]} E_t\left(\widehat{\Lambda}_{t+1}\right) + \\ &\quad + \frac{\gamma_2 R^{\gamma_2}(1-\delta)}{\left[\Lambda\frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2}\right]} E_t\left(\widehat{r}_{t+1}\right) \end{aligned} \tag{34}$$

therefore

$$\gamma_2 \widehat{r}_t \approx \widehat{q}_{t,t+1} + E_t\left(\widehat{\pi}_{t+1}\right) + \epsilon_1 E_t\left[\widehat{y}_{t+1} - \widehat{k}_{t+1} + \widehat{\Lambda}_{t+1}\right] + \epsilon_2 E_t\left(\widehat{r}_{t+1}\right) \tag{35}$$

$$\epsilon_1 = \frac{\Lambda\frac{\alpha}{K}Y}{\left[\Lambda\frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2}\right]}, \epsilon_2 = \frac{\gamma_2 R^{\gamma_2}(1-\delta)}{\left[\Lambda\frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2}\right]}$$

To simplify the elasticities in the above expression, we use that the steady state expression for capital:

$$R^{\gamma_2} = \beta\left[\Lambda\frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2}\right] \tag{36}$$

$$\frac{R^{\gamma_2}}{\beta} = \left[\Lambda\frac{\alpha Y}{K} + (1-\delta)R^{\gamma_2}\right] \tag{37}$$

$$\Lambda\frac{\alpha Y}{K} = R^{\gamma_2}\frac{(1-\beta(1-\delta))}{\beta} =$$

To simplify $\epsilon_2 = \beta\gamma_2(1-\delta)$,
$\epsilon_1 = (1-\beta(1-\delta))$

3b. Now, we compute the log-linearization of $\beta E_t \frac{(C_{t+1}^{-\sigma})}{(C_t^{-\sigma})\pi_{t+1}} = Q_{t,t+1}$

$$\widehat{q}_{t,t+1} = -\sigma \left(E_t \left(\widehat{c}_{t+1} - \widehat{c}_t \right) \right) - E_t \left(\widehat{\pi}_{t+1} \right) = -\widehat{r}_t$$

$$\begin{aligned} \gamma_2 \widehat{r}_t &\approx -\widehat{r}_t + E_t \left(\widehat{\pi}_{t+1} \right) + \\ & +(1-\beta(1-\delta)) E_t \left[\widehat{y}_{t+1} + \widehat{\Lambda}_{t+1} - \widehat{k}_{t+1} \right] + \beta \gamma_2 \left(1 - \delta \right) E_t \left(\widehat{r}_{t+1} \right) \end{aligned}$$

To compare this equation with the conventional NK model, $\gamma_2 = 0$, and we

$$0 \approx \widehat{q}_{t,t+1} + E_t \left(\widehat{\pi}_{t+1} \right) + (1-\beta(1-\delta)) E_t \left[\widehat{y}_{t+1} - \widehat{k}_{t+1} + \widehat{\Lambda}_{t+1} | \right]$$

$\widehat{q}_{t,t+1} = -\sigma \left(E_t \left(\widehat{c}_{t+1} - \widehat{c}_t \right) \right) - E_t \left(\widehat{\pi}_{t+1} \right) = -\widehat{r}_t$ than the NK interest rate will reflect the monopoly distortions ($\widehat{\Lambda}_{t+1}$) and the marginal cost of capital without the cost channel. Hence,

$$\left(\widehat{r}_t - E_t \left(\widehat{\pi}_{t+1} \right) \right) = (1-\beta(1-\delta)) E_t \left[\widehat{y}_{t+1} - \widehat{k}_{t+1} + \widehat{\Lambda}_{t+1} \right]$$

4. Log-linearization of lambda (18) :

$$\widehat{\Lambda}_t = \gamma_1 \widehat{r}_t + \widehat{w}_t - \widehat{p}_t + \widehat{h}_{i,t} - \widehat{y}_{i,t} \tag{38}$$

6 The Phillips curve:

$$\widehat{\pi}_t = \beta E_t \left(\widehat{\pi}_{t+1} \right) + \kappa \widehat{\Lambda}_t \tag{39}$$

where $\kappa = (1-\omega)(1-\omega\beta)/\omega$, and ω is the Calvo parameter. We can use the labour-share to measure the real marginal cost.$S_l = \frac{WH}{PY}$

$$\widehat{\Lambda}_t = \gamma_1 \widehat{r}_t + \widehat{w}_t - \widehat{p}_t + \widehat{h}_t - \widehat{y}_t = \gamma_1 \widehat{r}_t + \widehat{s}_l \tag{40}$$

7. Credit market:

$$\frac{m_{t+1}}{Y_t} - \frac{C_t}{Y_t} - \gamma_2 \frac{I_t}{Y_t} = \left(\gamma_1 - 1 \right) S_t \tag{41}$$

as $S_t = \frac{W_t H_t}{P_t Y_t}$. The Log-linearisation of that is given by

$$\begin{aligned} \left(\gamma_1 - 1 \right) \widehat{s}_t &= \frac{m/Y}{\varpi} \widehat{m}_{t+1} - \frac{s_c}{\varpi} \widehat{c}_t - \frac{\gamma_2 s_i}{\varpi} \widehat{i}_t - y_t \quad \text{for } \gamma_1 \neq 1 & (42) \\ \widehat{m}_{t+1} &= \frac{s_c}{(s_c + \gamma_2 s_i)} \widehat{c}_t + \frac{\gamma_2 s_i}{(s_c + \gamma_2 s_i)} \widehat{i}_t \quad \text{for } \gamma_1 = 1 & (43) \end{aligned}$$

where $\varpi = m/Y - (s_c + \gamma_2 s_i)$. Given the data on the velocity of money we set $\frac{m}{Y} = 1/0.44$ or the elasticity when $\gamma_2 = 1$ is $\epsilon_v = \frac{m}{m-Y} = \frac{\frac{m}{Y}}{\frac{m}{Y} - 1} = \frac{1/0.44}{1/0.44 - 1} = 1.785\,7$.

We can use the real marginal cost to measure the labour-share $S_t = \frac{WH}{PY}$:

$$\widehat{\Lambda}_t = \gamma_1 \widehat{r}_t + \widehat{w}_t - \widehat{p}_t + \widehat{h}_t - \widehat{y}_t = \gamma_1 \widehat{r}_t + \widehat{s}_t \quad (44)$$

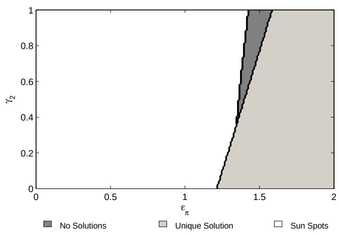
$$\widehat{s}_t = \widehat{\Lambda}_t - \gamma_1 \widehat{r}_t \quad (45)$$

8. Money Process

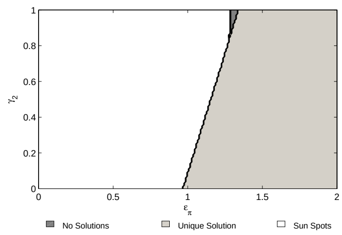
Nominal Money balances evolve as follows $M_{t+1} = M_t + V_t$. Dividing by P_t a and log linearizing gives (note that $m_t = \frac{M_t}{P_{t-1}}$):

$$\widehat{m}_{t+1} = \widehat{m}_t - \widehat{\pi}_t + v_t$$

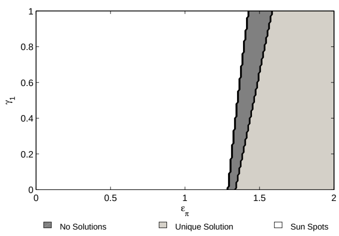
Figure 1: Cost Channel Effects - Varying γ_1 and γ_2



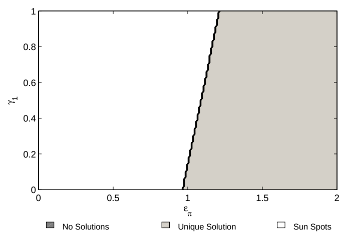
(a) Effect of Inv. Cost Channel - $\gamma_1 = 1$



(b) Effect of Inv. Cost Channel - $\gamma_1 = 0$

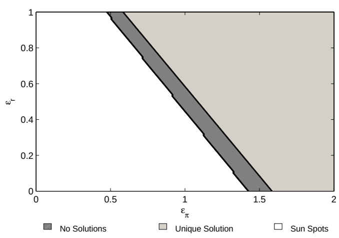


(c) Effect of Labour Cost Channel - $\gamma_2 = 1$

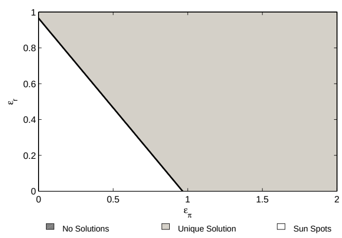


(d) Effect of Labour Cost Channel - $\gamma_2 = 0$

Figure 2: Cost Channel Effects - Interest Rate Smoothing ($\epsilon_y = 0.5$)

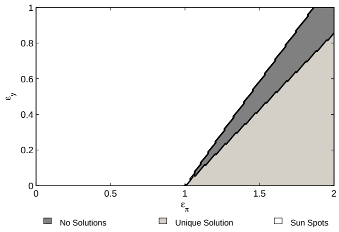


(a) Full Cost Channel

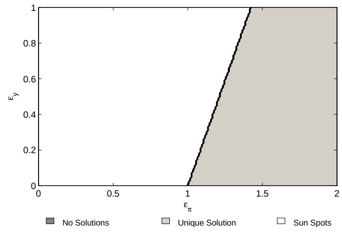


(b) No Cost Channel or NKM model

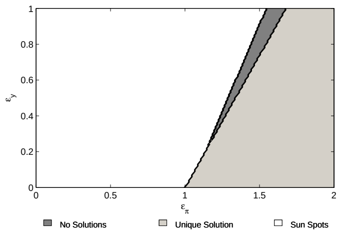
Figure 3: Cost Channel Effects - Output Targeting ($\epsilon_r = 0$)



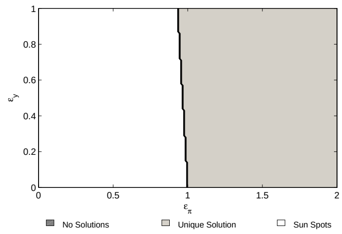
(a) Full Cost Channel



(b) Labour Cost Channel



(c) Investment Cost Channel



(d) No Cost Channel or NKM model

Figure 4: Impulse Responses - Policy Shock

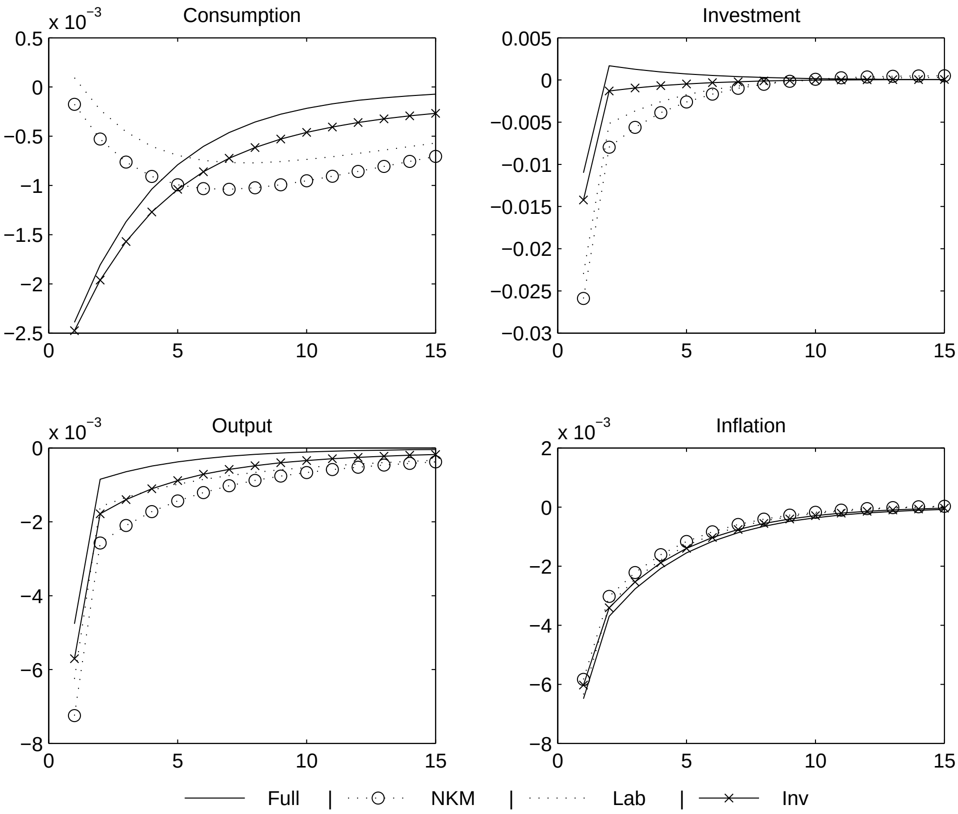


Figure 5: Impulse Responses - Taste Shock

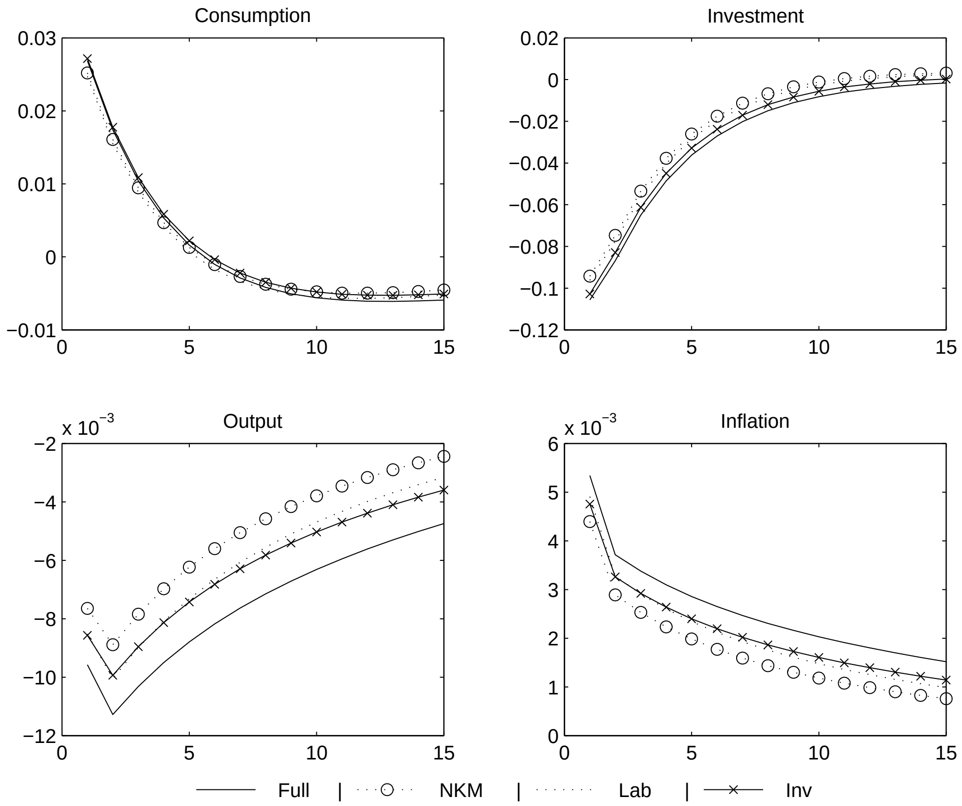


Figure 6: Impulse Responses - Investment Shock

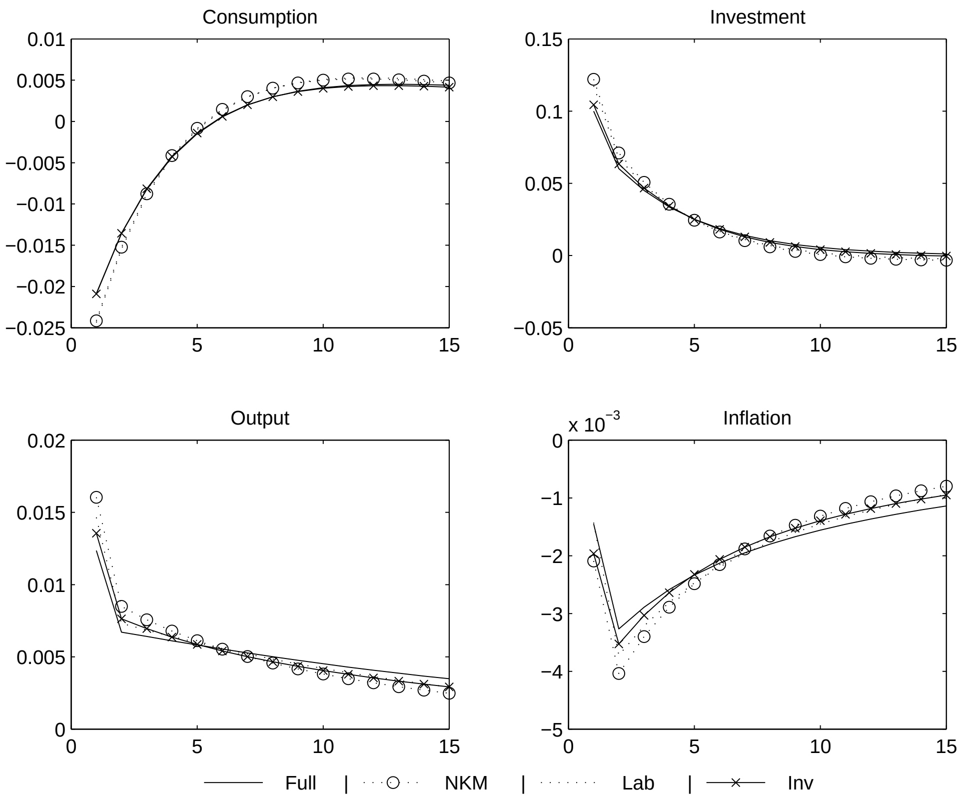


Figure 7: Impulse Responses - Inflation Shock

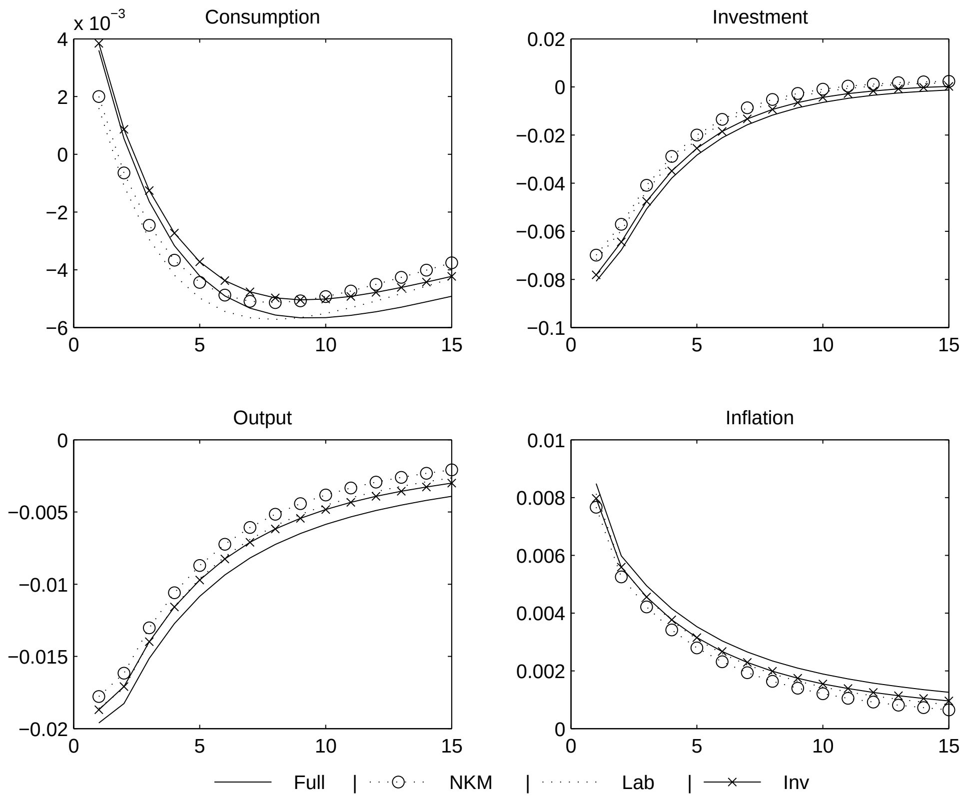


Figure 8: Liquidity Effects - Full Cost Channel